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ABSTRACT

This is part one of a three-part manual for teachers using SMSG junior high school text materials. Each chapter contains an introduction and a collection of sample text questions. Each section contains a discussion related to the topic at hand and answers to all the exercises. Chapter topics include: (1) rational numbers and coordinates; (2) equations; (3) scientific notation; (4) decimals, and the metric system; and (5) drawings and constructions. (MP)

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MATHEMATICS
STUDY GROUP

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MATHEMATICS FOR
JUNIOR HIGH SCHOOL
Commentary for Teachers
VOLUME II (Part 1)

(preliminary edition)

SE027968

MATHEMATICS FOR JUNIOR HIGH SCHOOL

Commentary for Teachers

Volume II (Part 1)

(preliminary edition)

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The preliminary edition of this volume was prepared at a writing session held at the University of Michigan during the summer of 1959, based, in part, on material prepared at the first SMSG writing session, held at Yale University in the summer of 1958. This revision was prepared at Stanford University in the summer of 1960, taking into account the classroom experience with the preliminary edition during the academic year 1959–60.

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PREFACE

Key ideas of junior high school mathematics emphasized in this text are: structure of arithmetic from an algebraic viewpoint; the real number system as a progressing development; metric and non-metric relations in geometry. Throughout the materials these ideas are associated with their applications. Important at this level are experience with and appreciation of abstract concepts, the role of definition, development of precise vocabulary and thought, experimentation, and proof. Substantial progress can be made on these concepts in the junior high school.

Fourteen experimental units for use in the seventh and eighth grades were written in the summer of 1958 and tried out by approximately 100 teachers in 12 centers in various parts of the country in the school year 1958-59. On the basis of teacher evaluations these units were revised during the summer of 1959 and, with a number of new units, were made a part of sample textbooks for grade 7 and a book of experimental units for grade 8. In the school year 1959-60, these seventh and eighth grade books were used by about 175 teachers in many parts of the country, and then further revised in the summer of 1960.

Mathematics is fascinating to many persons because of its opportunities for creation and discovery as well as for its utility. It is continuously and rapidly growing under the prodding of both intellectual curiosity and practical applications. Even junior high school students may formulate mathematical questions and conjectures which they can test and perhaps settle; they can develop systematic attacks on mathematical problems whether or not the problems have routine or immediately determinable solutions. Recognition of these important factors has played a considerable part in selection of content and method in this text.

We firmly believe mathematics can and should be studied with success and enjoyment. It is our hope that this text may greatly assist all teachers who use it to achieve this highly desirable goal.

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NOTE TO TEACHERS

Based on the teaching experience of over 100 junior high school teachers in all parts of the country and the estimates of authors of the revisions (including junior high school teachers), it is recommended that teaching time for Part 1 be as follows:

Chapter		Approximate number of days
1		15
2		15
3		13
4		10
	Total	53

Teachers are urged to try not to exceed these approximate time allotments so that pupils will not miss the chapters at the end of the course. Some classes will be able to finish certain chapters in less than the estimated time.

Throughout the text, problems, topics, and sections which were designed for the better students are indicated by an asterisk (*). Items starred in this manner should be used or omitted as a means of adjusting the approximate time schedule.

Chapter 1

RATIONAL NUMBERS AND COORDINATES

General Remarks

This chapter is intended to provide an informal introduction to negative rational numbers. The aim is to present the negative numbers as real, useful quantities which have already been used by the students for some time. The examples and illustrations have been selected to advance this point of view, and the teacher can develop enthusiastic interest by emphasizing applications of this sort.

Although the approach is informal, the important ideas about negative numbers have been included. With a few exceptions, the discussion of formal properties of the negative rationals has been placed in the exercises. It is expected that some classes (especially those who have done well on the Volume I SMSG material) will enjoy working on these formal aspects of the number system. The problems suggested in the exercises provide a good basis for such a treatment. On the other hand, classes without previous experience with SMSG texts may find these problems difficult at first. This will be especially true when the chapter is being taught at the beginning of the school year.

Our major aim here is to achieve a real understanding and familiarity with negative rationals without insisting upon complete mastery of formal techniques and details. The student should be encouraged to take time to figure things out by reference to the number line or to any of the other devices which have been introduced to make the negatives plausible to him.

The sections on coordinates and graphs capitalize on the use of the number line in introducing negative numbers. Section 1-9 which introduces the graphs of $y = ax + b$ and $y = x^2$ provides both application and practice in the multiplication of two negative numbers. For this reason the calculation of pairs of coordinates for such graphs should serve an especially useful purpose. With these thoughts in mind the teacher can guide the preliminary discussion of graphs in Section 1-5 to anticipate the need for multiplication of positive and negative numbers. This should provide excellent motivation for the discussion of multiplication in 1-6 and the later use of products in 1-9.

The teacher should think of this chapter in relation to both Chapter 2 on equations and to Chapter 6 on real numbers. Much can be done in the class discussion to anticipate the problems of these later chapters and to provide a background for their study.

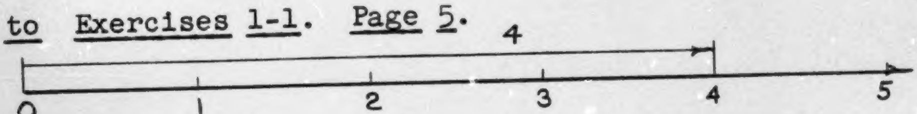
It is expected that 15 days class time should be ample for most classes to cover this chapter. Students who have just completed the first volume of SMSG material will probably be able to complete the chapter in a shorter time.

1-1. The Number Line.

Page 1. This discussion of the number line is a rather complete one intended to interest the student by relating the number to its associated point and to the corresponding directed line segment. Students who have studied Volume I of the SMSG Junior High School texts will probably be able to cover this section very rapidly. It provides a good review of material in Chapters 3, 6 and 8 of Volume I. Teachers who have not taught the SMSG seventh grade program will probably wish to read these three chapters (3, 6, 8) before teaching this material.

Answers to Exercises 1-1. Page 5.

1. a.



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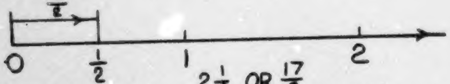

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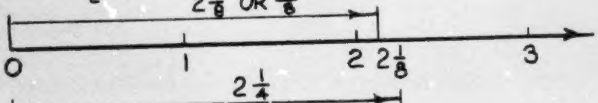

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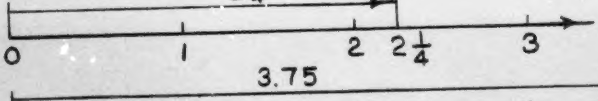
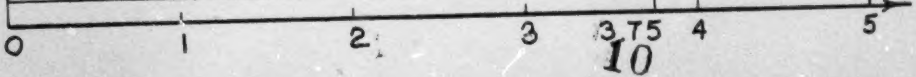
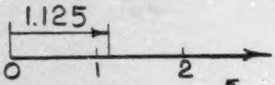


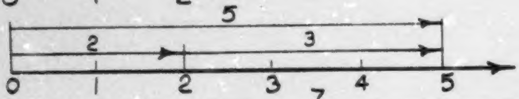
e.



f.



2. a.



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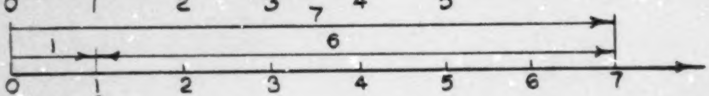
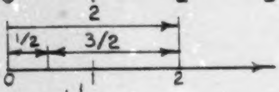

7
1
6
0
1
2
3
4
5
6
7

c.



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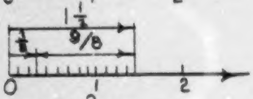

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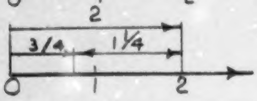

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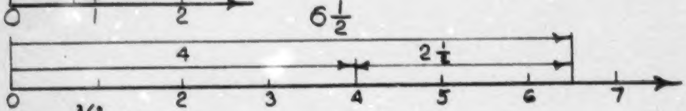
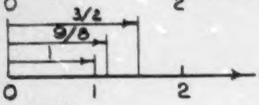
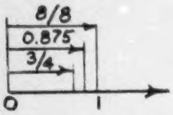

6 1/2
4
2 1/2
0
1
2
3
4
5
6
7

3. a.



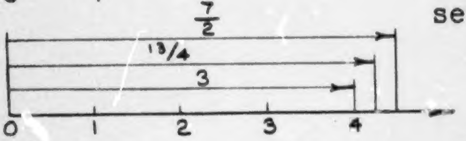
$\frac{3}{2}$ is the largest number in the set.

b.


8/8
0.875
3/4
0
1

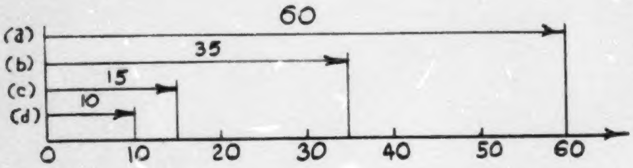
$\frac{8}{8}$ is the largest number in the set.

c.

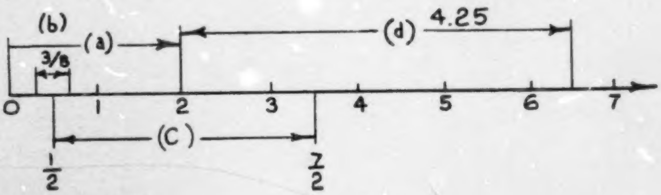

7/2
13/4
3
0
1
2
3
4

$\frac{7}{2}$ is the largest number in the set.

4.


60
35
15
10
(a)
(b)
(c)
(d)
0
10
20
30
40
50
60

5.


(b)
3/8
(a)
(d)
4.25
0
1
2
3
4
5
6
7
(C)
1/2
7/2

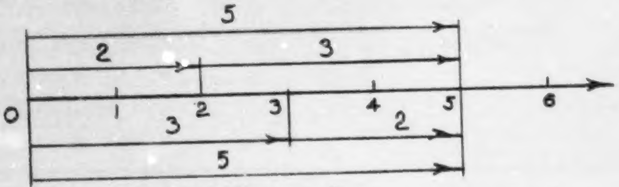
(a) [0,2], mid-point is 1

(b) [1/8, 5/8], mid-point is 3/8

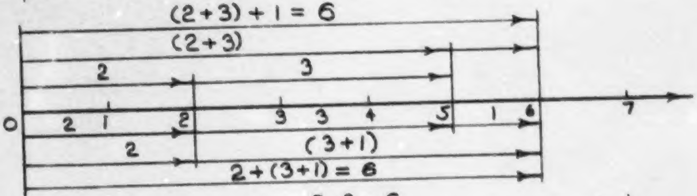
(c) [1/2, 7/2], mid-point is 2

(d) [2, 6.5], mid-point is 4.25

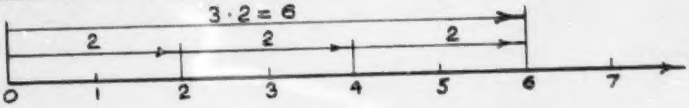
6.


5
2
3
0
1
2
3
4
5
6
3
2
5

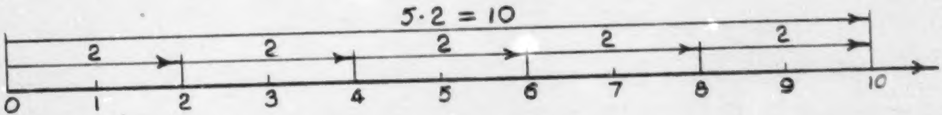
7.


(2+3) + 1 = 6
(2+3)
2
3
0
2
1
2
3
3
4
5
1
6
7
2
(3+1)
2+(3+1) = 6

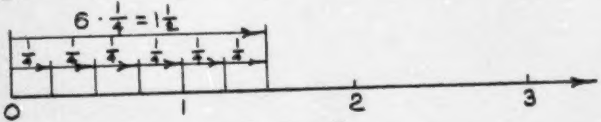
8. (a)


3·2 = 6
2
2
2
0
1
2
3
4
5
6
7

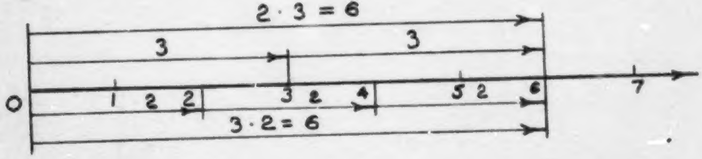
(b)


5·2 = 10
2
2
2
2
2
0
1
2
3
4
5
6
7
8
9
10

(c)


6 · 1/4 = 1 1/2
1/4
1/4
1/4
1/4
1/4
1/4
0
1
2
3

9.


2 · 3 = 6
3
3
0
1
2
2
3
2
4
5
2
6
7
3·2 = 6

This illustrates the commutative property of multiplication.

1-2. Negative Rational Numbers.

The negative numbers have been introduced with reference to the number line to provide geometric reality and an intuitive feeling for them. Discussions of their use in situations describing "opposite" characteristics should contribute also to a feeling of familiarity and usefulness. The idea of "opposite" is one deserving special emphasis.

The opposite characteristics should be stressed. The ordering of the rational numbers on the number line has been introduced and is worth continued discussion. It is valuable to develop a natural acceptance of the fact that "is less than" means the same as "precedes on the number line." To say that one number "is greater than" another means simply that the one "follows" the other in the number line.

Note that the idea of magnitude, which suggests the absolute value, of a number has been avoided. This idea is, of course, implicit in the length of the arrow associated with a rational number. It seems less troublesome, however, to avoid specific mention of absolute value in this informal treatment.

The student should be encouraged to provide sketches for the numbers with which he deals. To save his time and to focus attention on true understanding, he should be encouraged to draw a rough sketch which need not be carefully scaled. It is only important that relative lengths and relative positions should be faithfully represented. An intelligent sketch will often help the student's understanding more than a carefully drawn scale used in a routine way. In this section, and in the one which follows, the student is getting a valuable introduction to operations with directed line segments or vectors. Although not mentioned specifically, the notion of directed distances is being developed in this incidental way.

In this chapter a number like negative three is written as $^-3$, $^-(3)$, or $(^-3)$. The use of parentheses is more helpful in some contexts then in others. For example in

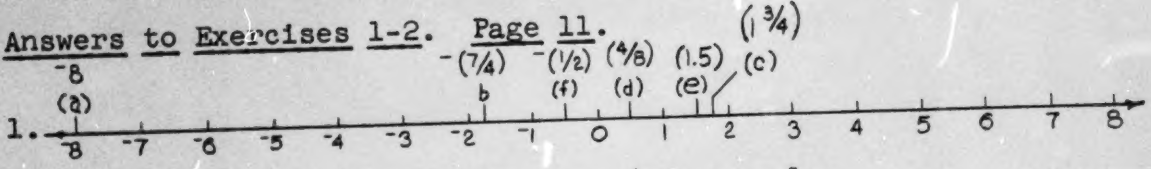
$$4 + (^-3) \quad \text{or} \quad 4 - (^-3)$$

the use of parentheses in this way should make the meaning more clear. With negative rational numbers which are not integers the authors recommend that the notation $^-(\frac{3}{4})$ be used exclusively until the study of division. Then students learn that

$$^-(\tfrac{3}{4}) = \frac{^-3}{4} = -\frac{3}{4}.$$

Answers to Exercises 1-2. Page 11.

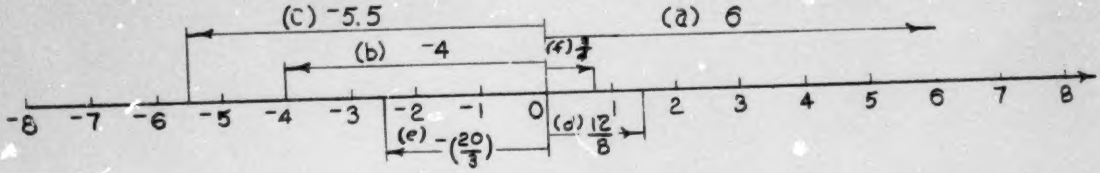
1.



$^-8$ (a)
$-(7/4)$ b
$^-(1/2)$ (f)
$(4/8)$ (d)
(1.5) (e)
$(1\frac{3}{4})$ (c)
$^-8$ $^-7$ $^-6$ $^-5$ $^-4$ $^-3$ $^-2$ $^-1$ 0 1 2 3 4 5 6 7 8

yes $^-(\frac{7}{4})$ and $1\frac{3}{4}$ $\frac{4}{8}$ and $^-(\frac{1}{2})$

2.



(c) $^-5.5$
(a) 6
(b) $^-4$
(f) $\frac{3}{4}$
(e) $-(\frac{20}{3})$
(d) $\frac{12}{8}$
$^-8$ $^-7$ $^-6$ $^-5$ $^-4$ $^-3$ $^-2$ $^-1$ 0 1 2 3 4 5 6 7 8

3. $^-6,\ ^-4,\ ^-(\frac{7}{4}),\ ^-(\frac{3}{8}),\ \frac{1}{4},\ \frac{5}{8},\ \frac{3}{4}$

largest is $\frac{3}{4}$

smallest is $^-6$

4. (a) $^+2000$
$^-6000$

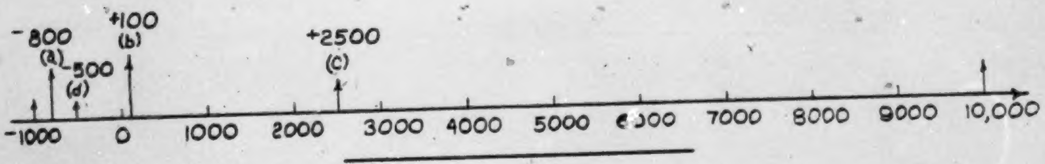
(b) $^+100$
$^-50$

(c) $^-15$
$^+10$

(d) $^+2$
$^-4$

5. $^-2,\ ^-1,\ 0,\ ^+1,\ ^+2,\ ^+3$

6.



-800 (a)
$+100$ (b)
-500 (d)
$+2500$ (c)
-1000 0 1000 2000 3000 4000 5000 6000 7000 8000 9000 10,000

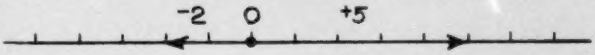
1-3. Addition of Rational Numbers.

Page 12. The treatment of addition continues the emphasis on the number line and the use of arrows to represent position and negative rationals. Here, especially, the student should be encouraged to sketch in a rough but essentially accurate fashion. A proper grounding at this stage should give the student something to fall back on in later periods of perplexity.

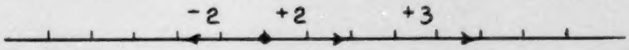
Our initial discussion of the use of arrows in the positive number line did not include a discussion of subtraction by means of arrows. Nor do we at this stage discuss the equivalence between the addition of ($^-3$) and the subtraction of ($^+3$). Unless the issue is unavoidably introduced into class discussion, it would seem preferable to avoid discussion of these matters at this stage.

The paragraph on addition inverses, while important, may be considered optional for those without the previous background of material in Chapter 12 of Volume I. It is important that teachers understand that the drawing on Page 20 is not intended to diagram the sum $5 + (^{-}2)$ in the same way as other sums were diagrammed earlier in this section. In this diagram the additive inverse relationship of 2 and $^{-}2$ is being emphasized. The net effect of adding $^{+}5$ and $^{-}2$ may be thought of as follows.

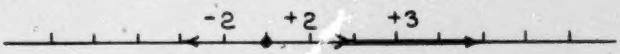
The $^{+}5$ and the $^{-}2$ are shown as arrows on the number line.



The number "5" may be thought of as $(^{+}2 + {}^{+}3)$.



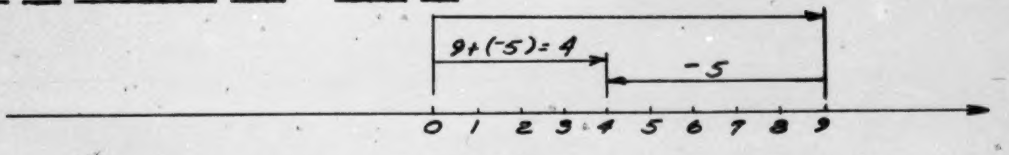
Since $^{+}2$ is the additive inverse of $^{-}2$, or the "true opposite" of $^{-}2$, we can picture the $^{+}2$ as an arrow equal in length to $^{-}2$, but in the opposite direction.



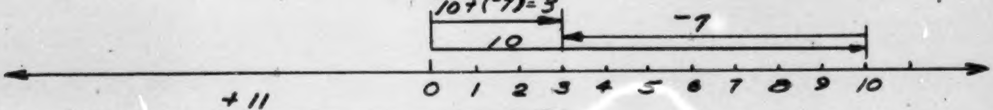
Since $(^{-}2) + (^{+}2) = 0$, the arrow representing the $^{+}2$ is, in effect, removed from the arrow representing the $^{+}5$. The remaining arrow, representing $^{+}3$, is the sum.

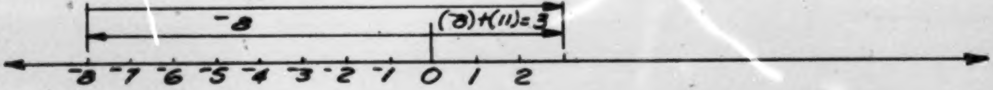
Answers to Exercise 1-3. Page 16.

1. (a)



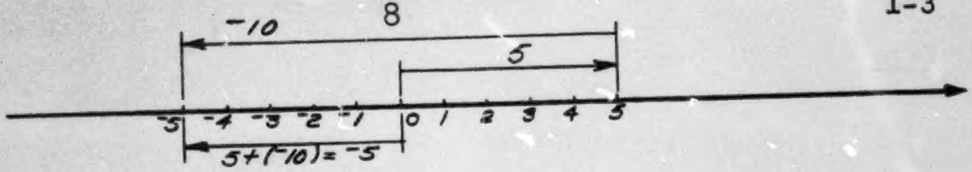
(b)



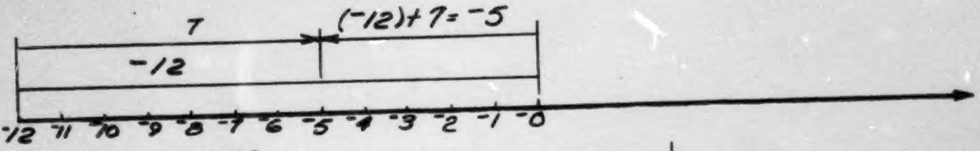


(c)

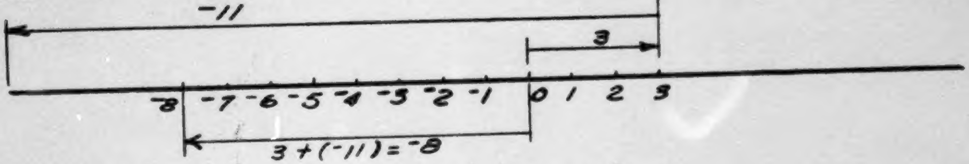
(d)


-10
8
5
-5
-4
-3
-2
-1
0
1
2
3
4
5
5+(-10)=-5

(e)


7
(-12)+7=-5
-12
-12
-11
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0

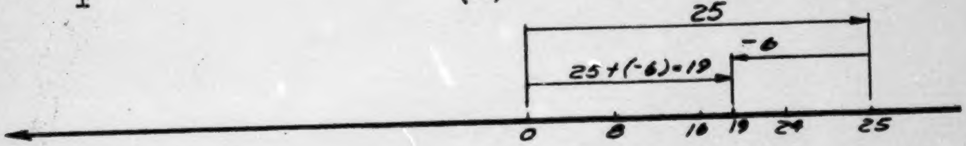
(f)


-11
3
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
3+(-11)=-8

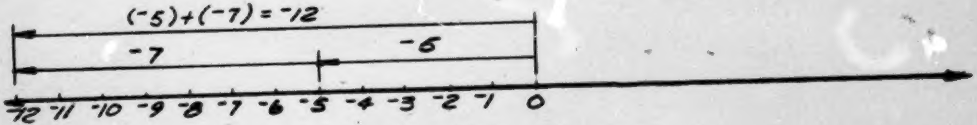
2. (a) 0
(b) 4
(c) $^{-}6$
(d) $^{-}1$
(e) 1/2
(f) 2/3
(g) 12
(h) 0

3\.

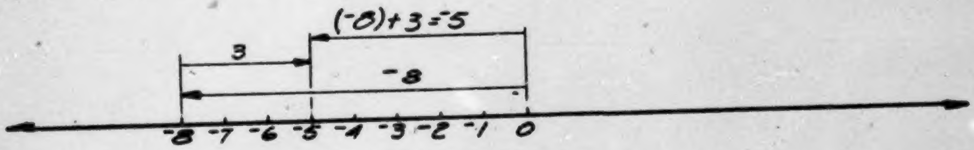
(a)


25
-6
25+(-6)=19
0
8
16
19
24
25

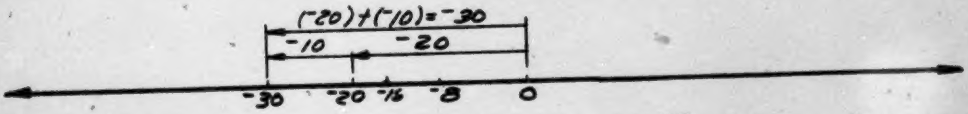
(b)


(-5)+(-7)=-12
-7
-6
-12
-11
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0

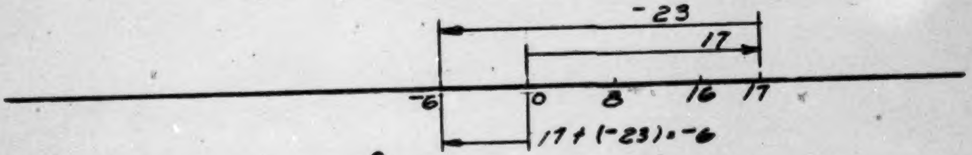
(c)


(-8)+3=-5
3
-8
-8
-7
-6
-5
-4
-3
-2
-1
0

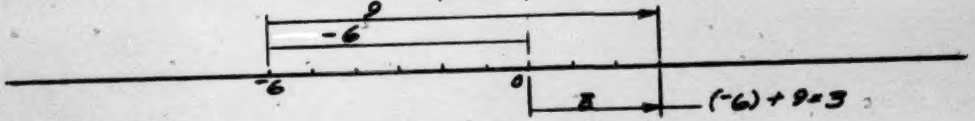
(d)


(-20)+(-10)=-30
-10
-20
-30
-20
-16
-8
0

(e)


-23
17
-6
0
8
16
17
17+(-23)=-6

(f)



4. (a) $^{-}5$
(b) $^{-}9$
(c) $^{-}11$
(d) $^{-}6$
(e) $^{-}8$
(f) $^{-}6$

Page 17.

5. (a) January $^{+}5000$ April $^{+}1000$
February $^{+}2000$ May $^{-}4000$
March $^{-}6000$ June $^{-}3000$

(b) $^{-}5000$ or \$ 5000 loss

(c) $^{+}1000$ or \$ 1000 profit

(d) $^{-}12,000$ or \$ 12,000 loss

6. (a) $^{+}4$ rate of boy upstream
$^{-}2$ rate of current downstream

(b) $^{+}2$ resultant rate of boy upstream

7. (a) $^{+}17$, $^{-}6$, $^{+}11$, $^{-}3$

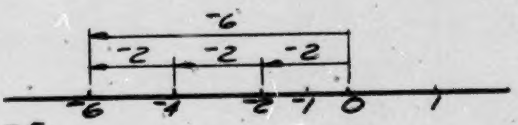
(b) $20 + 17 = 37$
$37 + (^{-}6) = 31$
$31 + 11 = 42$
$42 + (^{-}3) = 39$
ball is on the 39 yard line

(c) $17 + (^{-}6) + 11 + (^{-}3) = 19$ The net gain is 19 yards.

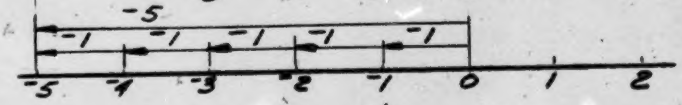
8. (a) $13,000 + 5000 + (^{-}3000) = 15,000$

(b) 15,000 ft.

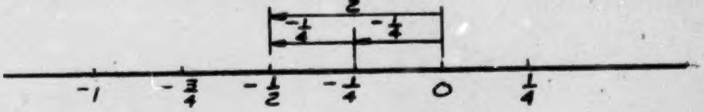
9. (a) $3(^{-}2) = {}^{-}6$



(b) $5(^{-}1) = {}^{-}5$



(c) $2 \cdot {}^{-}(\frac{1}{4}) = {}^{-}(\frac{1}{2})$.



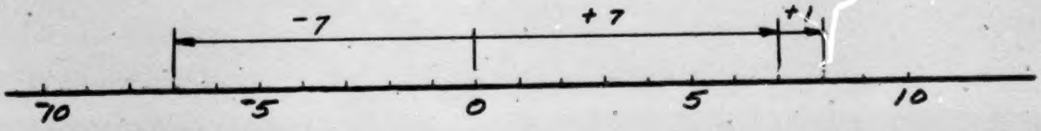
Answers to Class Exercises 1-3a.

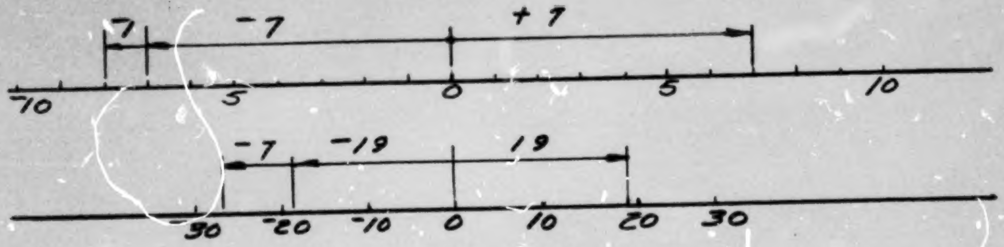
1. $^{-}7$, 9, $^{-}11$, 12, 6, $^{-}15$, 20, 0, $\frac{2}{3}$, $^{-}(\frac{4}{9})$, $\frac{7}{8}$, $^{-}(\frac{30}{31})$

2. The pairs in (c) and in (d) are additive inverses.

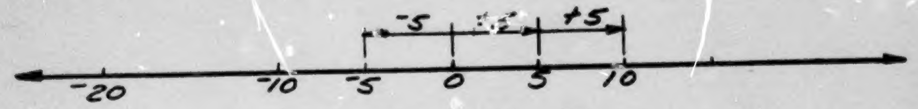
Answers to Class Exercises 1-3b. Page 18.

1.

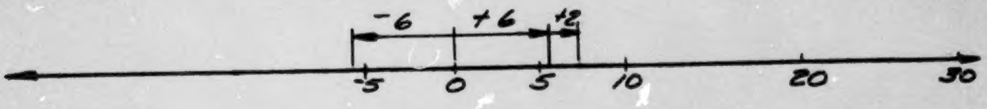




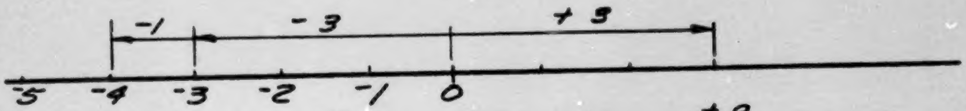
2. (a)



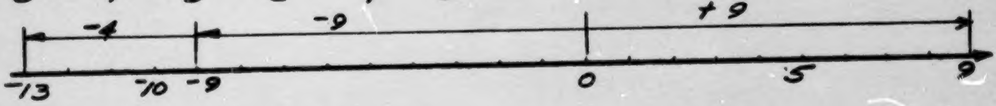
(b)



(c)



(d)



1-4 Coordinates.

Page 21. The use of a number line in introducing positive and negative rational numbers establishes, essentially, a system of coördinates on the line. The principal new idea here is the use of the term, coordinate, and the notation (2) to designate a coordinate of the point to which the number 2 has been assigned on the number line. Note that we have actually defined "a coordinate" rather than "the coordinate" although we have not made an issue of maintaining this distinction. A point on the line will have a different coordinate if the position of the origin is changed or if the unit of length is changed. Indeed, coordinate systems could be defined in which the scale, or method of assigning numbers to points, is not uniform. One might, for example, think of the scale on a slide rule as being a kind of number scale.

Credit for the invention of a coordinate system in a plane is often given to the French mathematician and philosopher, Rene Descartes. The method used by **Descartes** first appeared in a mathematical treatise published in 1637. More recently historians of science have pointed out that the essence of the idea appeared in the earlier work of several mathematicians. In any case, coordinate geometry is of considerably more recent origin than the Euclidean geometry of the traditional high school course.

Just as the number line provides for the association of a number with a point on a number line, a coordinate system in the plane provides for the association of a pair of numbers with a point in the plane. In the pair the numbers are ordered. There is a one-to-one correspondence between all rational number pairs and some of the points in the plane. In a coordinate system as described in this chapter, there are many points with which no ordered number pair is associated. An example is $(\sqrt{2}, \sqrt{2})$.

As with a coordinate of a point on a line we will speak of a pair of coordinates which is an ordered number pair, associated with a point in the plane.

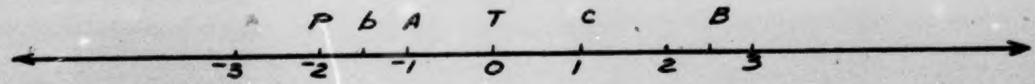
In this section students should learn (a) to locate the point when coordinates of the point are given, and (b) to give an ordered number pair associated with a point, when the point is given.

It is recommended that students be encouraged to use care in drawing the axes and labeling the scales on the axes. It is suggested that the numbering of the scale be below the X-axis and to the left of the Y-axis. It will help the students to keep in mind that the axes are lines, not line segments, if the axes are drawn with arrows as shown.

Always label the X-axis and the Y-axis as shown in the diagrams in the student's text, at least until students have had quite a bit of experience in plotting points and graphs of equations.

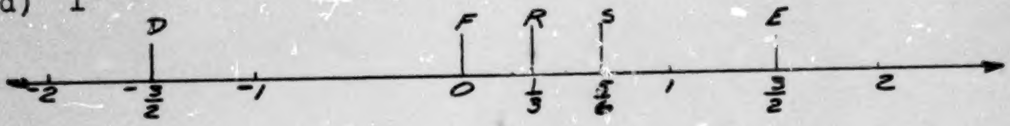
Answers to Exercises 1-4 (a). Page 22.

1.



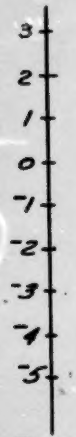
2. (a) 1 1/2"
   (b) 4 1/2"
   (c) 4"
   (d) 1"

3.


D
F
R
S
E
-2
-3/2
-1
0
1/3
5/6
1
3/2
2

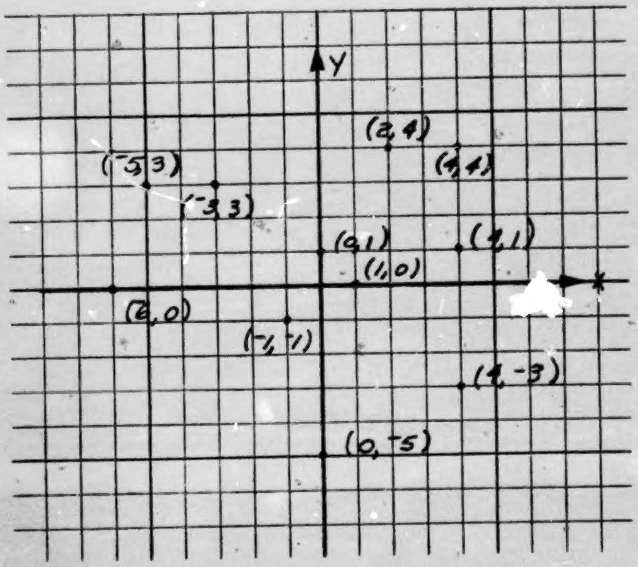
4. (a) 1/3 mile
   (b) 3 miles

5.

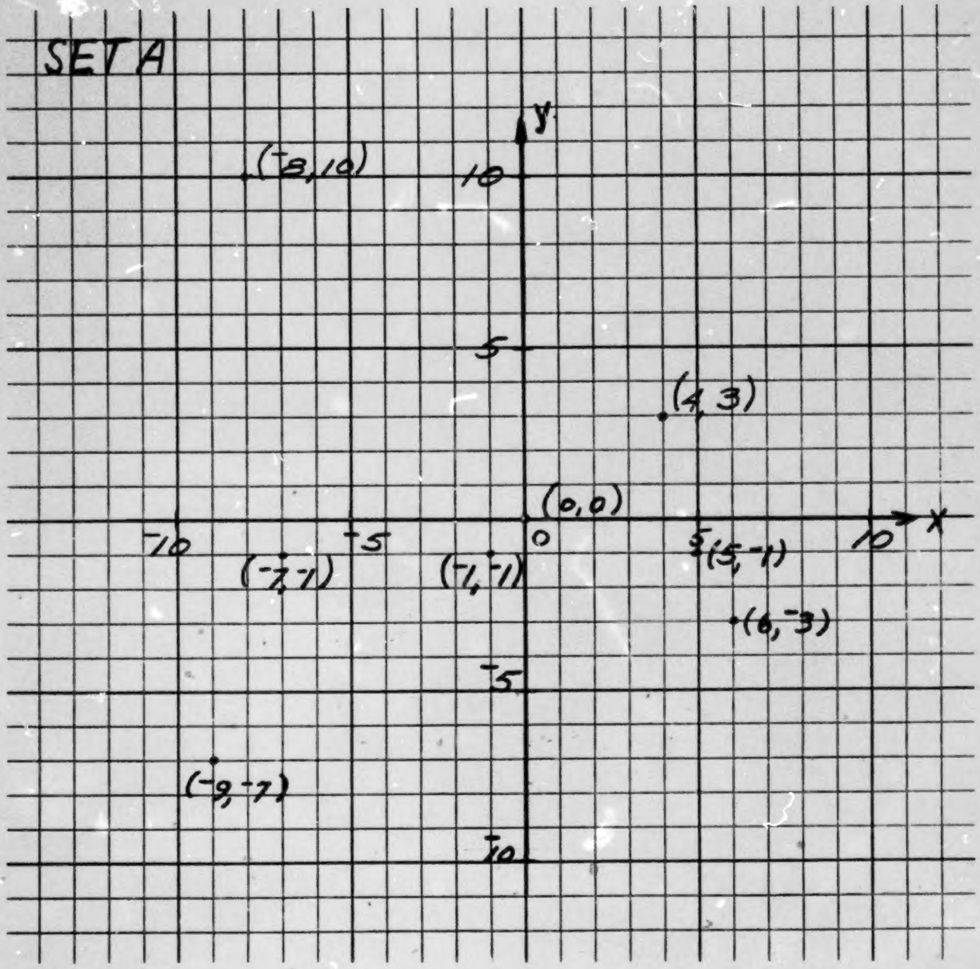


Answers to Exercises 1-4b. Page 26.

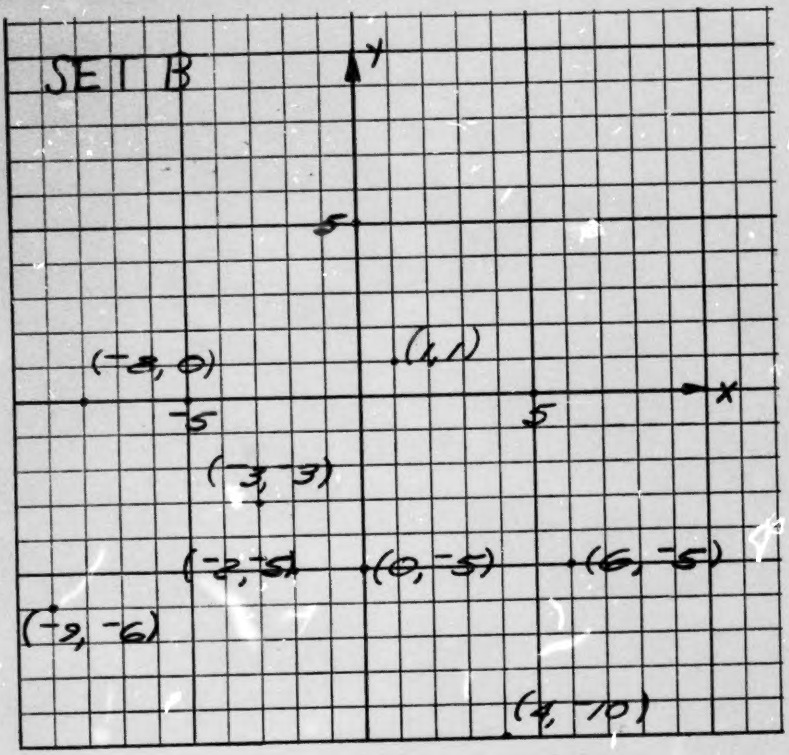
1.



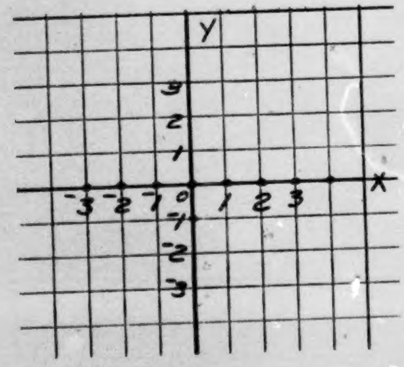
2.



2. con't



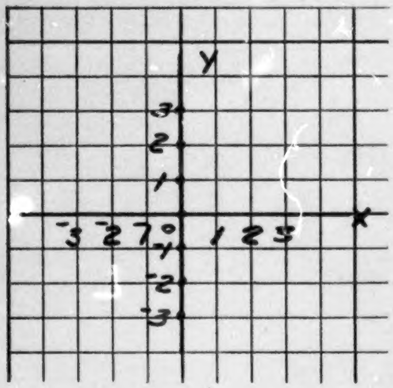
3. (a)



(b) yes

(c) yes

4. (a)



(b) yes

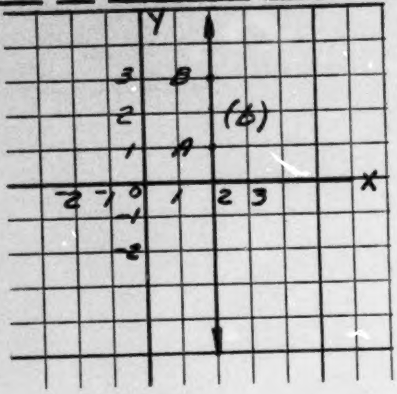
(c) yes

Answers to Class Exercises 1-4. Page 28.

1. (a) I
   (b) IV
   (c) II
   (d) III
   (e) I
   (f) IV
   (g) II
2. (a) I
   (b) III
   (c) II
   (d) III
3. (a) Y-axis
   (b) X-axis
   (c) At the origin
4. The quadrants are defined by the intersection of half-planes which do not contain the axes. Hence their intersection does not contain the axes.

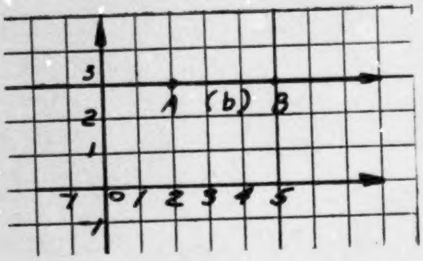
Answers to Exercise 1-4c. Page 29.

1.



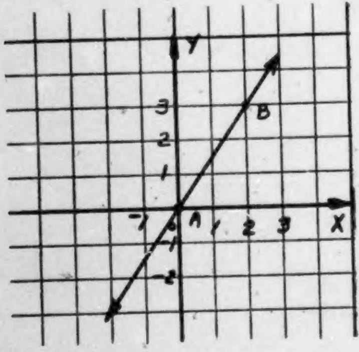
(c) the Y-axis

2.



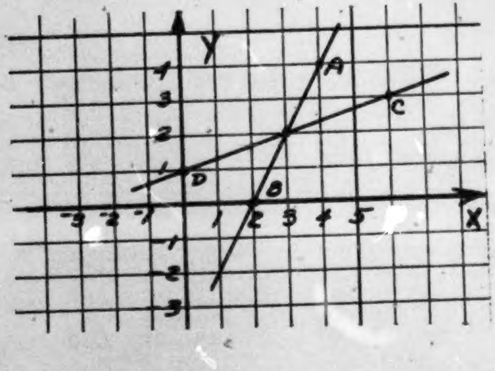
(c) the X-axis

3.



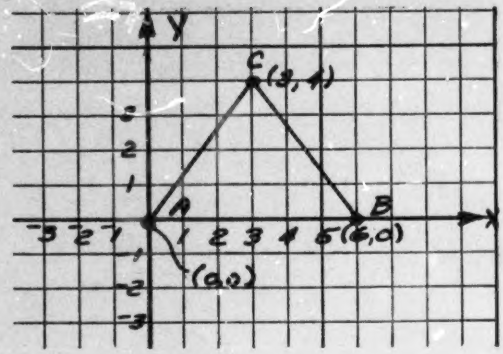
(c) neither axis.
AB is an oblique line.

4.



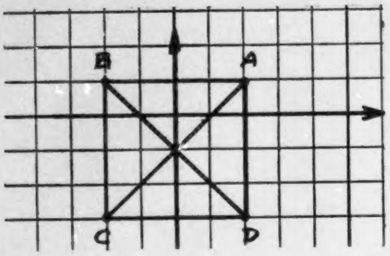
(e) {(3,2)}

5.



(c) isosceles

6.



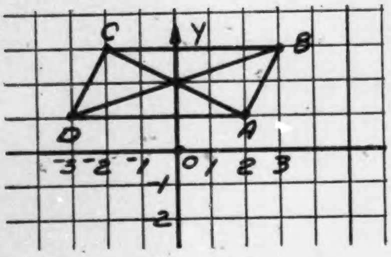
(c) yes

(e) $(0, {}^{-}1)$

7.

(c) parallelogram

(e) (0, 2)



1-5. Graphs.

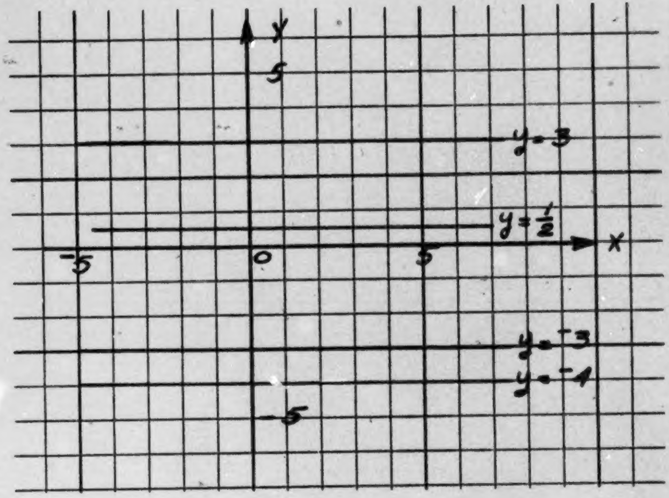
In sketching the graphs of conditions (expressed by equations), the students are carrying the association of geometric ideas (points and lines) and arithmetic ideas (numbers, number pairs, and equations) one step further. The use of a coordinate system in the plane provides for an association of a line (a set of points) with an equation and an equation with a set of points. In this section, only the simplest cases of equations are considered. Until students have learned more about multiplication and subtraction of rational numbers (both positive and negative) we are limited to conditions expressed by equations of the types $x = a$, $y = a$, $y = x + a$ and $x = y + a$, where a is a rational number. Even in this small beginning it is not too early for students to think about the two-way association of equation to line and line to equation. The exercises include equations of the form $y = x + a$, only for $a = 0$. The teacher may wish to ask students to sketch graphs of equations like $y = x + 2$ and $y = x + {}^{-}3$ at this time.

In this section we have chosen to talk about the graphs of a set of points described by a condition, such as $x = a$. Since properties of equations will be considered in Chapter 2, it has seemed better to talk about conditions on the coordinates of points, such as $y = x$, in an introductory treatment.

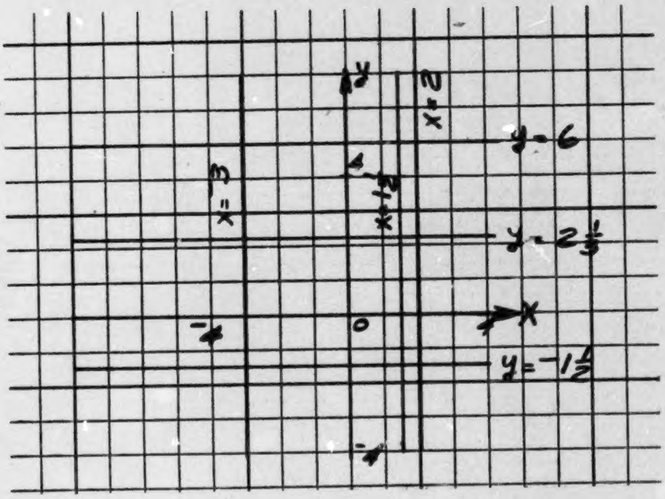
In addition to graphing equations of lines, pupils should be able to graph some inequalities. The graphs of inequalities in this section are all half planes. The teacher may wish to review ideas associated with half planes in the seventh grade course. It should present no special difficulty to pupils to recognize that all points above the line of equation, $y = 3$, have coordinates which satisfy the inequality $y > 3$.

Answers to Class Exercises 1-5. Page 31.

1. (a) $y = 3$
   (b) $y = {}^-3$
   (c) $y = {}^-4$
   (d) $y = \frac{1}{2}$

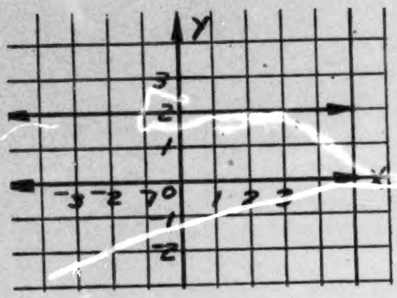


2.

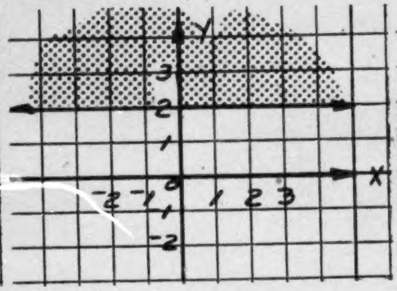


Answers to Exercises 1-5a. Page 36.

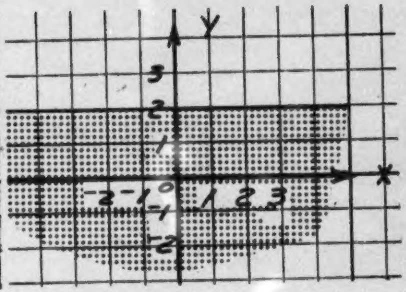
1. (a) $y = 2$



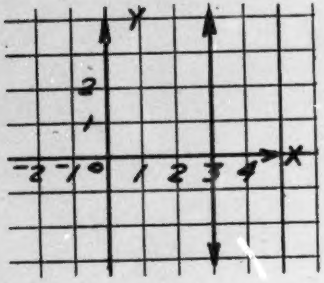
(b) $y > 2$



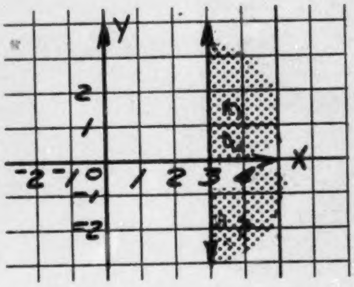
(c) $y < 2$



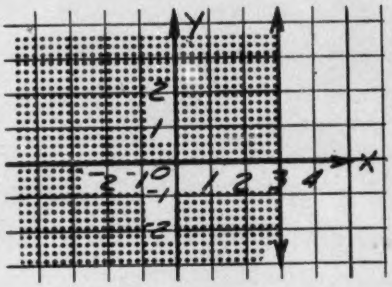
(d) $x = 3$



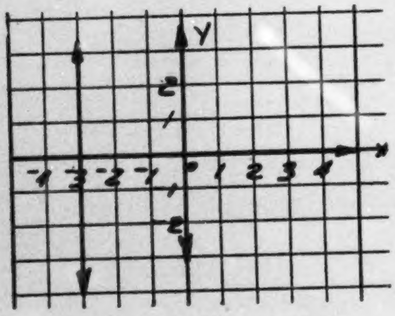
(e) $x > 3$



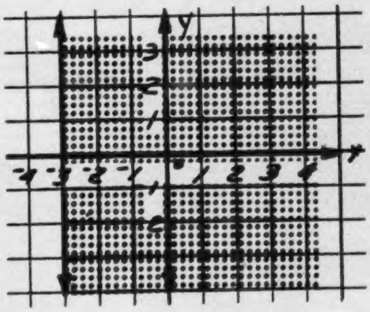
(f) $x < 3$



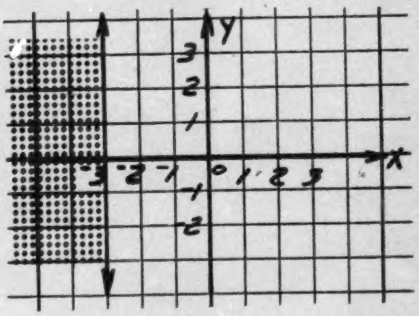
(g) $x = {}^-3$



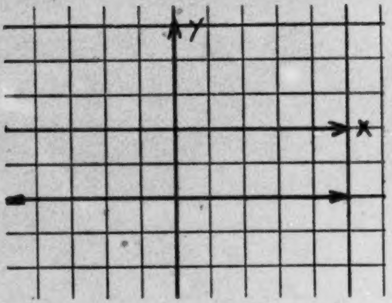
(h) $x > {}^-3$



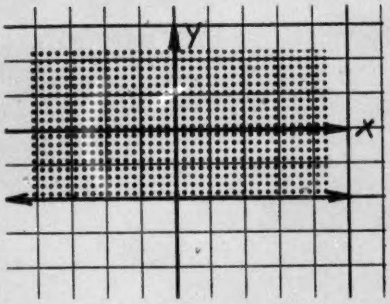
(i) $x < {}^-3$



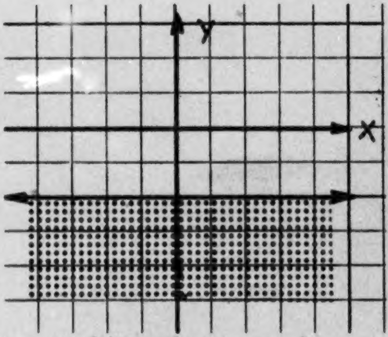
(j) $y = {}^{-}2$



(k) $y > {}^{-}2$



(l) $y < {}^{-}2$



(m) the graph of (a) is on the line two units above and parallel to the x-axis.
the graph of (b) is on the half-plane above the graph of $y = 2$.
the graph of (c) is on the half-plane below the graph of $y = 2$.

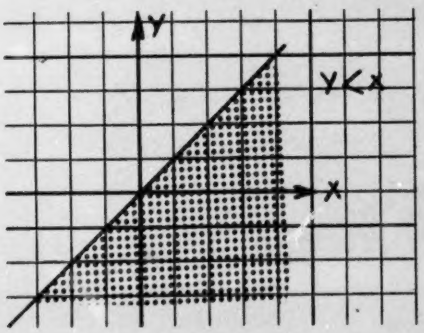
(n) the graph of (d) is on the line three units to the right of the y-axis.
the graph of (e) is on the half-plane to the right of the graph of $x = 3$.
the graph of (f) is on the half-plane to the left of the graph of $x = 3$.

Answers to Exercises 1-5b. Page 34.

1. (a) Possible points include $(4, 4)$, $({}^{-}4, {}^{-}4)$, $(5,5)$ $(5, {}^{-}5)$, $(\frac{1}{2}, \frac{1}{2})$, $(\frac{-1}{2}, \frac{-1}{2})$.

   (b) The graph of $y = x$ is in the text. The point here is to encourage the student to sketch a careful graph of it himself.

2. (a)



   (b) A possible set R is:
   $R : \{(0, {}^{-}1), (1,0), ({}^{-}1, {}^{-}3), (2, 1)\}$

3. The graph of $y = x$ is a line which passes through the origin and makes an angle of 45° with the x-axis.

   The graph of $y > x$ is the half-plane above and to the left of $y = x$. The graph of $y < x$ is the half-plane below and to the right of $y = x$.

1-6. Multiplication of Rational Numbers.

Page 35. In this chapter the treatment of all topics is intended to be quite informal. Ideas are related to earlier study by the students so that conclusions may appear to the student to be reasonable ones. There is absolutely no proof of the conclusions. In multiplication, students are asked to recall properties of a set of multiples of a whole number or of a multiplication table. Here are several examples: $5 \cdot 0 = 0$; $5 \cdot 1 = 5$; $5 \cdot 2 = 10$; $5 \cdot 3 = 15$; $5 \cdot 4 = 20$; etc. Each product is 5 less than the one which follows it. Hence, it is reasonable to conclude that if the number by which 5 is multiplied decreases by 1, the product decreases by 5. If we use integers to the left of 0, in order, the products can be expected to be $^-5$, $^-10$, $^-15$, etc. This leads us to the conclusion that $5 \cdot {}^-1 = {}^-5$, $5 \cdot {}^-2 = {}^-10$, $5 \cdot {}^-3 = {}^-15$, etc. When products like these are put in table form, by a similar argument we can make a conclusion about the product of two negatives.

This approach enables us to avoid the difficulties involved with negative multipliers, whether or not the number line is used. There is, however, no objection to the use of a number line approach (indeed, this appeared in Section 1-3 in the exercises) either as a supplementary method or as the principal method.

Teachers may also find it helpful to give other interpretations, such as, gains and losses in games, and running a film backward.

Full advantage should be taken of the opportunity to provide motivation for multiplication in pointing out that our ability to plot graphs of equations will be seriously limited unless we can find products of which at least one factor is a negative number.

The informal treatment cannot be expected to provide for complete mastery of this topic. It is an introduction to a topic which will be useful in Chapter 2 and other parts of the course, and particularly in later work in mathematics. Pupils should be allowed sufficient time to do the work. Most students will not be able to give products automatically after studying this chapter.

There may be more routine "drill" exercises than the teacher will wish to assign. Some of these, particularly the last two, Exercises 19 and 20, may be used for class discussion.

Answers to Exercises 1-6. Page 38.

1. The products increase in the 1-row, 2-row, and 3-row as we move to the right.
2. The products decrease, as we move down, in the $^{-}2$ column and the $^{-}1$ column.
3. $^{-}28, {}^{-}21, {}^{-}14, {}^{-}7, 0, 7, 14, 21, 28, 35, 42$
4. $20, 16, 12, 8, 4, 0, {}^{-}4, {}^{-}8, {}^{-}12, {}^{-}16, {}^{-}20$
5.

	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	
$^{-}5$	15	10	5	0	$^{-}5$	$^{-}10$	
$^{-}4$	12	8	4	0	$^{-}4$	$^{-}8$	
$^{-}3$	9	6	3	0	$^{-}3$	$^{-}6$	
$^{-}2$	6	4	2	0	$^{-}2$	$^{-}4$	
$^{-}1$	3	2	1	0	$^{-}1$	$^{-}2$	
0	0	0	0	0	0	0	
1	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	

6. (a) $(^{-}2)(1) = {}^{-}2$, $(1)(^{-}2) = {}^{-}2$, therefore $(^{-}2)(1) = (1)(^{-}2)$
   (b) $(^{-}3)(0) = 0$, $(0)(^{-}3) = 0$, therefore $(^{-}3)(0) = (0)(^{-}3)$
   (c) $(^{-}4)(^{-}5) = 20$, $(^{-}5)(^{-}4) = 20$, therefore $(^{-}4)(^{-}5) = (^{-}5)(^{-}4)$
   (d) $(15)(^{-}6) = {}^{-}90$, $(^{-}6)(15) = {}^{-}90$, therefore $(15)(^{-}6) = (^{-}6)(15)$

7. $(^-2)\,[(^-1)(5)] = (^-2)(^-5) = 10$, $[(^-2)(^-1)]\,(5) = (2)(5) = 10$,
therefore $(^-2)[(^-1)(5)] = [(^-2)(^-1)]\,(5)$

8. (a) $^-4\,(3 + 8) = (^-4)(3) + (^-4)(8) = (^-12) + (^-32) = {}^-44$
$^-4(3 + 8) = {}^-4(11) = {}^-44$
therefore $^-4(3 + 8) = {}^-4(11)$

(b) $^-2(^-3 + 6) = (^-2)(^-3) + (^-2)(6) = 6 + (^-12) = {}^-6$
$^-2(^-3 + 6) = (^-2)\,(3) = {}^-6$
therefore $^-2\,(^-3 + 6) = (^-2)(3)$

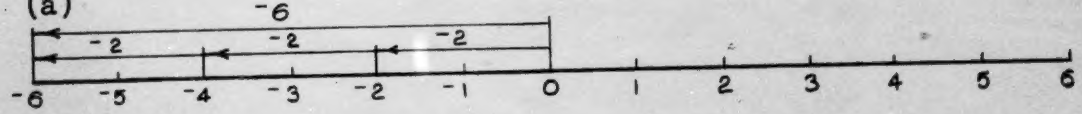
(c) $^-10[(^-8) + (^-1)] = (^-10)(^-8) + (^-10)(^-1) = 80 + 10 = 90$
$^-10[(^-8) + (^-1)] = {}^-10(^-9) = 90$
therefore $^-10[(^-8) + (^-1)] = (^-10)(^-9)$

9. (a) 0
(b) $^-8$
(c) $^-20$
(d) $^-24$
(e) $^-34$
(f) $^-245$
(g) 54
(h) 600
(i) 903
(j) 0.84
(k) $^-60$
(l) 576
(m) $^-66$
(n) $^-(\frac{160}{3})$
(o) $^-16$

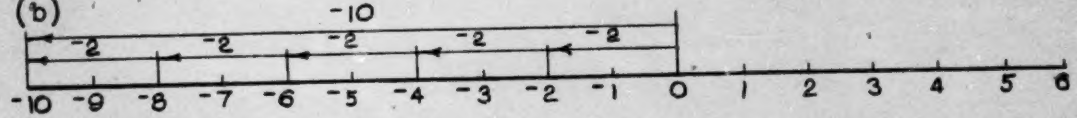
10. (a) $^-4$ (b) $^-5$ (c) $^-11$ (d) $^-8$ (e) $^-77$

11. $^-6 = (6)(^-1)$ by definition
$6(^-1) = (^-1)(6)$ because of the commutative property of multiplication;
therefore $^-6 = (^-1)(6)$ because of the transitive property of equality.

12. (a)


-6
-2
-2
-2
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6

(b)


-10
-2
-2
-2
-2
-2
-10
-9
-8
-7
-6
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-3
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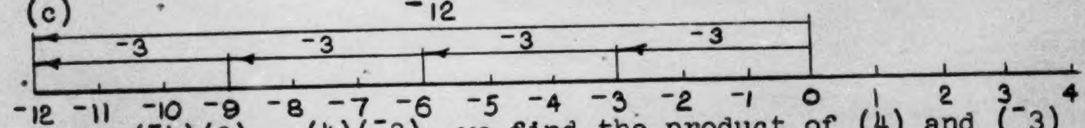
-1
0
1
2
3
4
5
6

(c)


-12
-3
-3
-3
-3
-12
-11
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4

13. Since $(^-4)(3) = (4)(^-3)$, we find the product of (4) and $(^-3)$ on the number line. After we find this product on the number line we claim it equals $(^-4)(3)$ because of the commutative property of multiplication.

14. (a) 39 yard line
(b) $45 + [(3)(^-2)]$ or $45 + (^-6)$

15. (a) 36 yard line
(b) In order to ascertain the initial position of the two teams we have to undo the addition of $3(^-2)$.

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16. (a) A football team has the ball on the 47 yard line. It loses 5 yards on each of 4 successive plays. Its new position is ___?___.
(b) After losing three yards on two successive plays a football team is on the 15 yard line. At what position was the team before these losses?

17. $^-9$

18. (a) $^-12$
(b) $^-15$
(c) $^-5$
(d) $^-10$
(e) 4
(f) 2

19. (a) $^-2$
(b) $^-3$
(c) $^-10$
(d) $^-4$
(e) 4
(f) $^-10$
(g) $^-1$
(k) 0
(i) $^-1$
(j) $^-6$
(h) $^-9$
(l) $^-6$
(m) 9
(n) $^-2$
(o) 10
(p) 2

20. (a) 60
(b) 12
(c) 15
(d) 42
(e) 23
(f) 300
(g) 40
(h) $^-40$
(i) $^-40$
(j) 42
(k) 60
(l) 110
(m) $^-192$
(n) 192
(o) 135
(p) $^-75$
(q) 0
(r) $^-16$
(s) 1000
(t) $^-60$
(u) 60
(v) 60

(w) $^-6$
(x) 16
(y) $^-27$
(z) 8

1-7. Division of Rational Numbers. Page 42.

In an informal introduction to negative numbers it seems appropriate to think of division only in terms of the inverse operation, multiplication. Similarly subtraction, by the additive method is the development chosen for this chapter. In the work of the remainder of this course the student will rarely, if ever, find it necessary to divide or subtract, using negative numbers. Mastery of division and subtraction can well be delayed until the first-year algebra course. After studying this section and the next, however, the pupil should feel that he knows how to find the answers to such questions as, "What is the quotient of $^-5$ and 2?" and "What is the number x for which $8 - (^-7) = x$?" even though his response may be by no means automatic.

The teacher should make full use of the opportunity to relate multiplication and division and to review some of the basic ideas about rational numbers developed in Chapter 6 of Volume I. It is recommended that emphasis be placed on interpreting the question, "What is $^-6 \div {}^-2$?", as another way of asking, "By what number should $^-2$ be multiplied to obtain $^-6$?" Frequent early reference might well be made to what is known from arithmetic. We know $15 \div 3 = 5$ because $5 \cdot 3 = 15$.

It is also recommended that the equation be used extensively in this development. There are two points to be emphasized.

Find n if $^-5n = {}^-35$.

a. Since we know $^-5 \cdot 7 = {}^-35$, $n = 7$ in
$$^-5n = {}^-35.$$

b. If
$$^-5n = {}^-35$$
$$n = \frac{^-35}{^-5}$$ by our extended definition of rational numbers to include negative numbers.

$$\frac{^-35}{^-5} = 7, \text{ since } {}^-5 \cdot 7 = {}^-35.$$

In exercises 1-7, Problems 8 through 15 provide for a development of a generalization about conditions under which the quotient is a negative or a positive number (or under which a fraction with negative numerator or denominator is a numeral for a negative number). It may be desirable to use these exercises for class discussion.

The teacher should notice that we do not talk about the "sign of a number" throughout the chapter. "Negative eleven" is a number, just as 193 is a number; $^-11$ is not a number with a sign attached.

Negative three-fourths $^-(\frac{3}{4})$ is a number. As a number it does not have a numerator and a denominator.

We may talk about the numerator and denominator of a fractional numeral for this negative number, such as $\frac{(^-3)}{4}$, or $\frac{3}{(^-4)}$.

In the last paragraph of this section there is one example involving the reciprocal of a negative number. This example is used as further illustrative material. The idea of reciprocal, itself, is not of special importance here. In mathematics, of course, we are interested always in the number, a, by which another number, b, must be multiplied to obtain the product, 1. The numbers a and b may be called multiplicative inverses, although we have not used that term in this chapter.

Answers to Exercises 1-7. Page 44.

1.	(a)	$^-28$	(d)	$^-72$	(g)	$^-3$
	(b)	$^+12$	(e)	72	(h)	4
	(c)	$^-12$	(f)	735	(i)	$\frac{2}{5}$
2.	(a)	$^-4$	(d)	$^-24$	(g)	$^-(\frac{3}{4})$
	(b)	$^-4$	(e)	$^-8$	(h)	$^-10$
	(c)	$^-6$	(f)	$^-21$	(i)	$^-(\frac{12}{25})$

3.

x	4	2	0	$^-2$	$^-4$	$^-5$	$^-6$
y	2	1	0	$^-1$	$^-2$	$^-2\frac{1}{2}$	$^-3$

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4.

x	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$^-(\frac{1}{2})$	$^-(\frac{3}{2})$
y	$^-8$	$^-6$	$^-4$	$^-2$	0	2	6

5. (a) $r = 5\frac{2}{3}$
(b) $r = {}^-2$
(c) $r = {}^-3$
(d) $r = 7$
(e) $r = \frac{3}{16}$
(f) $r = {}^-(\frac{1}{2})$
(g) $r = {}^-1\frac{1}{7}$
(h) $r = 1$
(i) $r = {}^-(\frac{1}{12})$

6. $P' = \{\frac{1}{6}, \frac{2}{3}, 1, \frac{6}{5}, {}^-1, {}^-(\frac{2}{3}), {}^-(\frac{3}{7})\}$

7. (a) 2
(b) $^-5$
(c) $^-5$
(d) 5
(e) $^-5$
(f) 5
(g) 5
(h) 3
(i) $^-36$
(j) 21
(k) $^-22$
(l) 13
(m) $^-4$
(n) $^-25$
(o) 0
(p) 13
(q) $^-12$
(r) $^-6$

8. (a) $n = \frac{(^-2)}{3} = {}^-(\frac{2}{3})$, (b) $n = \frac{2}{(^-3)} = {}^-(\frac{2}{3})$

9. $\frac{(^-2)}{3} = \frac{2}{(^-3)}$

10. (a) $n = \frac{^-6}{7} = {}^-(\frac{6}{7})$, (b) $n = \frac{6}{^-7} = {}^-(\frac{6}{7})$

11. $(^-6)\ n = 7$, $6n = {}^-7$.

12. (a) $n = (\frac{92}{25}) = 3\ \frac{17}{25}$
(b) $n = \frac{92}{25} = 3\ \frac{17}{25}$
(c) $n = \frac{1}{3}$
(d) $n = \frac{1}{3}$

13. $92 \div 25$, $^-92 \div {}^-25$

14. $(25)n = 92$ $^-25n = {}^-92$

15. (a) $bx = a$
(b) either both positive or both negative
(c) negative, positive

1-8. Subtraction of Rational Numbers.

Page 46. Some pupils will have first studied subtraction by the additive method but these pupils are almost certainly in the minority. Nevertheless all pupils should be familiar, from arithmetic, with the relationship involved and its importance. By the additive method in order to subtract 8 from 17 we think of what number added to 8 is 17. If the teacher uses this approach in this chapter, some review, using numbers from arithmetic will be desirable.

The explanation of subtraction, using the number line, is omitted because it seems a little too complicated for an informal treatment. There is no objection to the use of this idea by the teacher if he feels it contributes to understanding on the part of pupils. In Problem 5 the student is asked to suggest a method for using the number line in subtraction. Some teachers may wish to omit this problem while others may choose to make much more of it.

The problem about the boys playing a game is introduced as one "practical" application. Good teachers may think of many others, although real applications are scarce at this level. If students understand what they are doing, sufficient motivation will be provided by their curiosity about the process of subtraction as necessary for "completion" of the treatment of a new kind of number.

The term additive inverse is first introduced in 1-3 and is used again in Exercises 7 through 9. The teacher may choose to use additive inverses more extensively in class discussion. While the use of this idea is often helpful to students as a tool in obtaining correct answers, if this term is introduced too early in the study of subtraction its use may become a mechanical device to avoid thinking. It is not at all essential in the chapter that the student make such generalizations as "to subtract $^{-}12$ is the same as adding 12."

Answers to Exercises 1-8. Page 48.

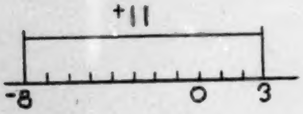
1. (a) 3
   (b) 0
   (c) $^{-}7$
   (d) 5
   (e) 10
   (f) 8
   (g) $^{-}45$
   (h) $^{-}67$
   (i) $^{-}28$

2. $\frac{11}{4}$, $\frac{11}{4} - {}^{-}(\frac{3}{4}) = \frac{7}{2}$, $\frac{11}{4} - \frac{7}{2} = {}^{-}(\frac{3}{4})$

3. (a) $x = {}^{-}3$
   (b) $x = 11$
   (c) $x = {}^{-}11$
   (d) $x = 15$
   (e) ${}^{-}(\frac{13}{6})$
   (f) ${}^{-}(\frac{3}{2})$
   (g) 5
   (h) ${}^{-}(\frac{49}{12})$

4. (a) $^{-}5$
   (b) 3
   (c) $^{-}6$
   (d) 6
   (e) 3
   (f) 3
   (g) $\frac{1}{4}$
   (h) ${}^{-}(\frac{1}{2})$

5.



Is one wishes to jump from $^{-}8$ to $^{+}3$ one has to travel in a positive direction and one must cover 11 units
$^{+}3 - (^{-}8) = {}^{+}11.$

6. (a) $^{-}6$
   (b) $^{-}5$
   (c) 11
   (d) 2
   (e) ${}^{-}(\frac{6}{4}) = {}^{-}(\frac{3}{2})$
   (f) 2
   (g) $\frac{41}{3}$
   (h) $\frac{117}{3}$

7. (a) $^{-}10$ (b) 100 (c) ${}^{-}(\frac{1}{2})$ (d) ${}^{-}(\frac{7}{9})$ (e) $\frac{8}{5}$ (f) $\frac{49}{51}$

8. If $a - b = c$, and $a + (^{-}b) = c$, then $a - b = a + (^{-}b)$ (the transitive property of equality.)

9. (a) $(^{-}4) + (^{-}2)$
   (b) $(^{-}6) + (1)$
   (c) $8 + (3)$
   (d) $(^{-}11) + (13)$
   (e) ${}^{-}(\frac{5}{4}) + {}^{-}(\frac{1}{4})$
   (f) ${}^{-}(\frac{3}{2}) + (\frac{7}{2})$
   (g) $\frac{35}{3} + (2)$
   (h) $\frac{75}{7} + (6)$

10. (a) $^-7$
(b) $^-2$
(c) 4
(d) 10
(e) $^-10$
(f) $^-6$
(g) $^-11$
(h) 12
(i) 2
(j) 12
(k) $^-7$
(l) 11
(m) $^-7$
(n) 13

11.

x	$^-1$	0	1	2	3	4
y	$^-5$	$^-3$	$^-1$	1	3	5

12.

x	$^-2$	$^-1$	0	1	2	3	4
y	5	3	1	$^-1$	$^-3$	$^-5$	$^-7$

1-9. Graphs of Other Relations. Page 51.

Additional experience in plotting graphs of conditions or relations, expressed by equations and inequalities, will be of great value to the students in their study of Chapter 2. This experience will also provide for review of the work with graphs in 1-4 and 1-5, and with multiplication, division, and subtraction of rational numbers. The study of graphs is in itself one of the most important parts of secondary school mathematics.

The relations studied in this section are for the most part, expressed by equations of the form, $y = ax + b$, where a and b are rational numbers. Students should be able to plot graphs of equations of this form with accuracy when they have completed the chapter. The teacher should, however, allow sufficient time for their work especially since we are not expecting them to be able to find a number like $^-2x - 7$ for $x = {}^-3$ with the speed expected in first-year algebra.

The graph of the relation, $y = x^2$, is introduced as a problem so that the students will not begin to expect all graphs to be straight lines. Teachers may wish to use this problem as a class exercise, especially since part of the solution is included in the student text.

In equations of the type, $y = ax + b$, substitutions for x should be made. After the study of equations in Chapter 2, the students will be much better able to substitute a number for y and determine x.

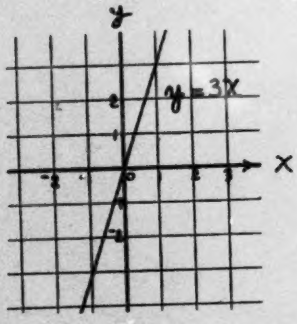
If students understand the graph of $y = 2x + 5$, then it should seem natural to them to expect that points above this line have coordinates which satisfy the inequality $y > 2x + 5$. The students should find it more convincing to substitute the coordinates of several points above the line to make sure that the inequality, in each case, after substitution for x and for y, is a true statement.

It may also assist students in understanding graphs of inequalities if they will substitute a value for x in the inequality; for example $x = 2$ in $y > 2\,(2) + 5$, $y > 9$. Then they should plot 3 or 4 points with x-coordinate equal to 2 and with y-coordinate greater than 9.

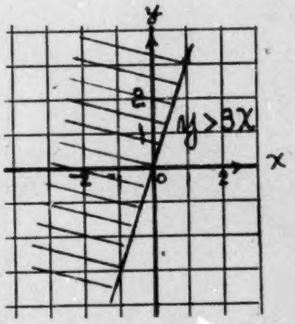
Answers to Exercises 1-9. Page 55.

1. sample set $W = \{(0,0), (1,3), (^-1,3), (2,6), (^-2,^-6)\}$

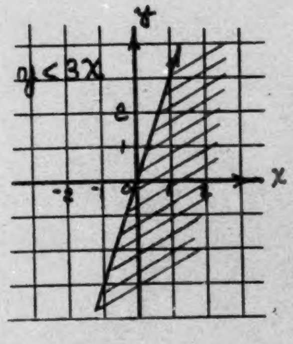
2.



3.



4.

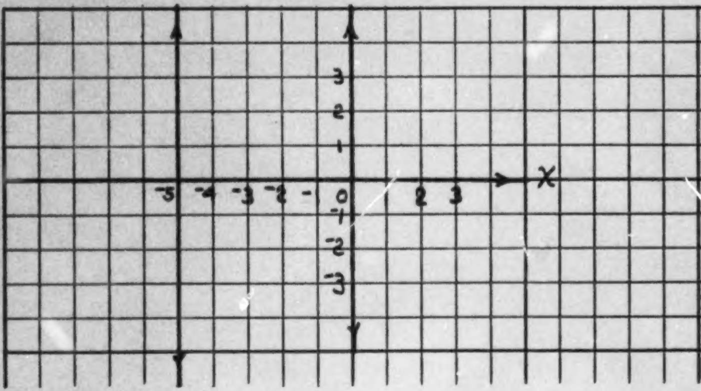


5. Yes. All points in the plane will be included in the graph since for each value of x, y is allowed to be less than $3x$, greater than $3x$, or equal to $3x$. Note: Strictly speaking these graphs will be not contain all points of the plane unless we have the real numbers to use as coordinates.

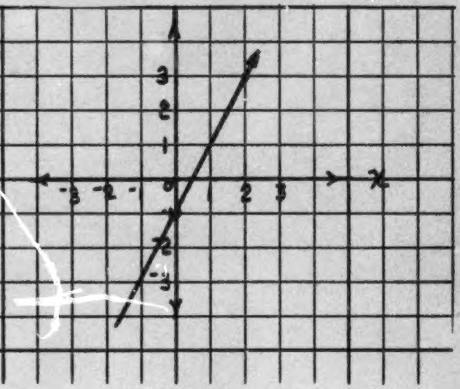
6. Sample points in the graph of $y > 3x + 2$ might be: $(0,3)$, $(1,6)$, $(2,9)$, $(3,11)$, $(4,14)$, $(^-1,^-2)$, $(^-2,^-5)$, $(^-3,^-8)$.

7. (a) $y > 3x + 2$
   (b) $y > 3x + 2$
   (c) $y > 3x + 2$
   (d) $y < 3x + 2$
   (e) $y > 3x + 2$

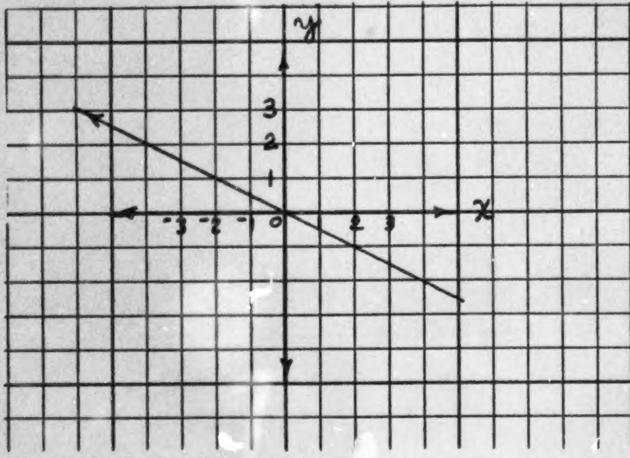
8. (a) $x = {}^{-}5$



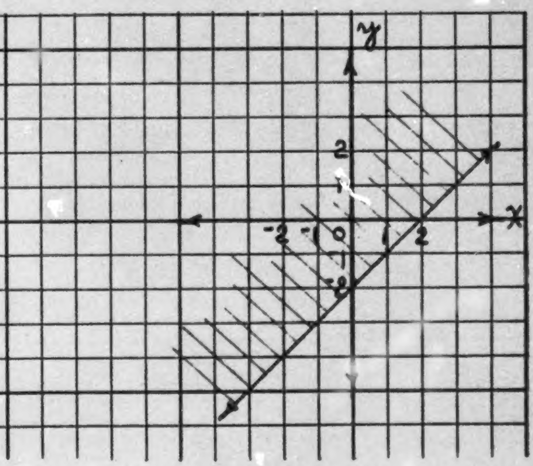
(b) $y = 2x - 1$



(c) $y = {}^{-}(\frac{1}{2})x$



(d) $y > x - 2$



9. (a) no
   (b) See Diagram (a) in the student text
   (c) yes
   (d) yes
   (e) Samples (5,25), $({}^{-}5,25)$, (6,36), etc.
   (f) yes

Sample Questions

True or False: Read each of the following statements. Decide whether each statement is True or False. If the statement is true, write "True" in the space provided. If a statement is false, write "False" in the space provided.

True 1. We can associate rational numbers with points on a line.

False 2. The point named by the ordered pair (3,2) is the same as the point named by (2,3).

False 3. The point $(^{-}4, ^{-}1)$ is located in Quadrant II.

True 4. The sum of a positive two and a negative two is zero.

True 5. There is no greatest number on the number line.

False 6. All negative rational numbers are associated with points on the number line to the right of zero.

True 7. The point on the number line associated with $^{+}2$ and the point on the number line associated with $^{-}2$ are equidistant from the point on the number line associated with 0.

False 8. The rational number $^{-}3$ is greater than the rational number $^{-}2$.

True 9. The graph of a condition is the set of points in the coordinate plane described by that condition.

False 10. The product and quotient of two negative numbers is a negative number.

Completion: Complete each of the following statements by supplying the word or words which makes the statement a true statement. Place your answers in the spaces provided at the left of each statement.

$^{-}7$ 1. The y-coordinate in the ordered pair $(3, ^{-}7)$ is ____

3 rd 2. The ordered pair $(^{-}4, ^{-}2)$ names a point in the coordinate plane located in the ________ quadrant.

$^{-}2$ 3. The sum of $^{+}4$ and $^{-}6$ is ________.

$^{+}10$ 4. The difference $(^{+}4) - (^{-}6)$ is ________.

$^{-}24$ 5. The product of $^{+}4$ and $^{-}6$ is ________.

$^{-}(\frac{2}{3})$ 6. The quotient of $(^{+}4) \div (^{-}6)$ is ________.

graph 7. The set of points in the coordinate plane described by a given condition is called the ________ of that condition.

10 8. If $^{-}5$ is subtracted from $^{+}5$, the difference is ________.

opposites or additive inverses 9. On a number line, the rational numbers assigned to two points equally distant from the origin are called ________ of each other.

right 10. On a number line, $^{-}6$ is greater than $^{-}7$ because it is assigned to a point which is to the ________ of the point to which $^{-}7$ is assigned to.

Multiple Choice: Read each statement carefully. Select the letter of the answer you think is correct. Place the letter in the answer space provided at the left of each statement.

A 1. Numbers which we associate with points on the number line to the left of 0 are called:
(A) Negative numbers (B) Positive numbers

B 2. Each point in the coordinate plane has how many numbers associated with it?
(A) one (B) two

B 3. The sum of $^{-}3$ and $^{-}6$ is
(A) $^{+}9$ (B) $^{-}9$

A 4. The difference $(^{-}3) - (^{-}6)$ equals:
(A) $^{+}3$ (B) $^{-}3$

B 5. The product of $^{-}3$ and $^{-}6$ is:
(A) $^{-}18$ (B) $^{+}18$

A 6. The quotient of $(^{-}3) \div (^{-}6)$ equals:
(A) $^{+}(\frac{1}{2})$ (B) $^{-}(\frac{1}{2})$

A 7. $4 \cdot [1 - 2]$ equals:
(A) $^{-}4$ (B) $^{+}4$

B 8. The difference $(^{+}3) - (^{+}2)$ is the same as:
(A) $^{+}3 - (^{-}2)$ (B) $^{+}3 + (^{-}2)$

B 9. The point in the coordinate plane named by the ordered pair $(3, ^{-}3)$ is located in
(A) Quadrant III (B) Quadrant IV

A 10. The graph of the condition $y > x$ lies:
(A) in a half-plane (B) on a line

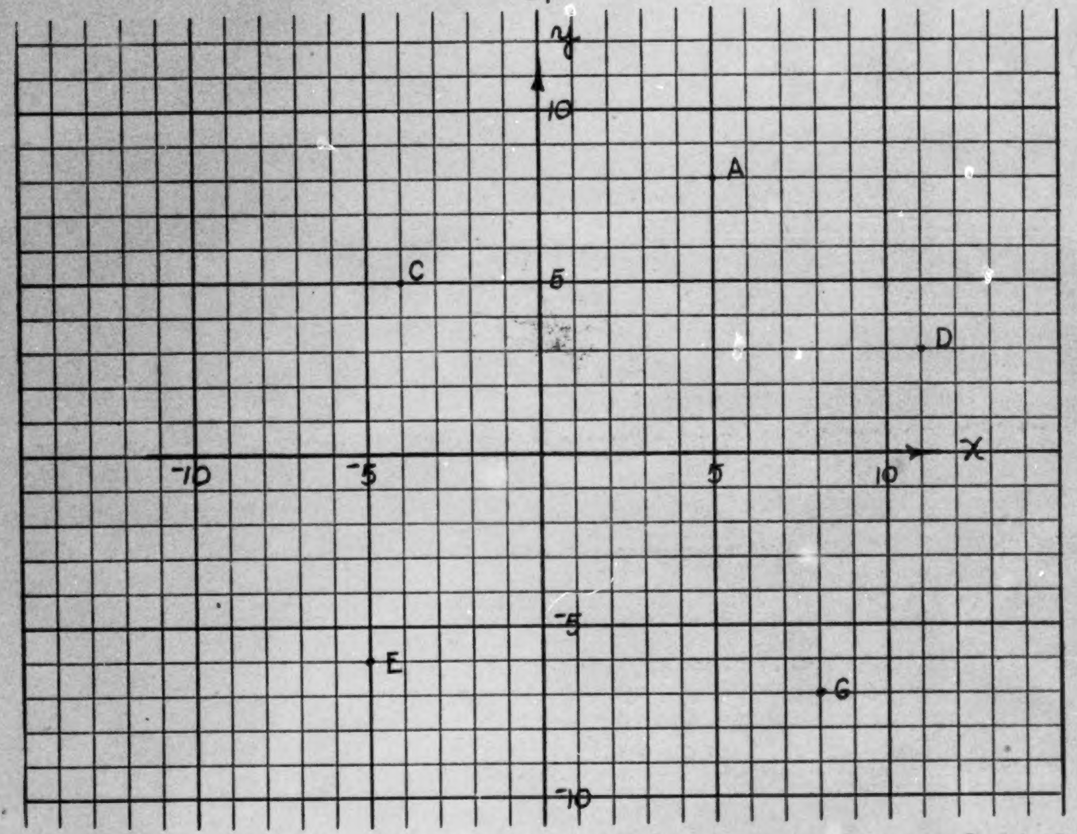
Matching: Study the items in Column A. Select the items from Column B which are best matched with the items listed in Column A. Place the letter of your choice from Column B in the answer spaces next to the items in Column A.

Column A

h 1. The sum of $^{-}6$ and $^{+}3$
j 2. The difference: $(^{-}6) - (^{+}3)$
k 3. The product of $(^{-}6)$ and $(^{+}3)$
i 4. The quotient $(^{-}6) \div (^{+}3)$
g 5. A point located in Quadrant II
f 6. An ordered pair described by the condition $y = 2x$
a 7. The graph of $y > x$
c 8. The opposite of $^{-}3$
d 9. The sum of a number and its opposite
b 10. The product of 3 and $\frac{1}{3}$

Column B

(a) points on a half plane
(b) 1
(c) $^{+}3$
(d) 0
(e) points on a line
(f) (2, 4)
(g) $(^{-}3, ^{+}4)$
(h) $^{-}3$
(i) $^{-}2$
(j) $^{-}9$
(k) $^{-}18$
(l) 18
(m) (4, 2)
(n) $^{-}1$



In the diagram above points A through B have been plotted. Select the ordered pairs from column b which are correctly associated with the plotted points.

Column a	Ans.		Column b
A (,)	(4)	1.	(¯5, ¯6)
B (,)	(7)	2.	(¯4, 5)
C (,)	(2)	3.	(¯3, 4)
D (,)	(8)	4.	(5, 8)
E (,)	(1)	5.	(4, ¯5)
F (,)	(12)	6.	(¯11, 3)
G (,)	(10)	7.	(3, ¯4)
		8.	(11, 3)
		9.	(10, ¯12)
		10.	(8, ¯7)
		11.	(5, 6)
		12.	(¯10, 12)

Sample Questions

1. k, ℓ, m, and n are rational numbers.
   (a) $k + \ell = m$. To what number is $m - \ell$ equal?
   (b) $k \cdot \ell = m$. Express k in terms of ℓ and m.
2. Supply the missing number to make a true statement.
   (a) $4 + ({}^-5) + (\quad) = 0$
   (b) ${}^-13 + (\quad) + 17 = 0$
   (c) $(\quad) + ({}^-13) + ({}^-4) = 0$
   (d) ${}^-8 \cdot (\quad) = {}^-16$
   (e) ${}^-15 - (\quad) = 8$
3. Sketch a number line and mark the points associated with the numbers
   $(\frac{1}{2})$, 6, ${}^-3$, $\frac{9}{2}$, and $\frac{{}^-4}{3}$
4. The average of five numbers is obtained by adding the numbers and dividing the sum by 5. Find the average of the numbers: ${}^-8$, ${}^-3$, ${}^-1$, 0, 10.
5. Complete the table for
   $y = {}^-3x$

x	${}^-2$	${}^-1$	0	1	
${}^-3x$					${}^-12$

6. Use 5 substitutions for x to graph the equation $y = 2x - 3$.
7. Graph the inequality $x > {}^-2$.
8. At the end of a game consisting of 5 rounds of play, Bill's score is 23 and Jack's score is ${}^-15$. What is the difference in their scores?
9. The vertices of a trapezoid are at the points $({}^-4,3)$, $(0,0)$, $(3,0)$, and $(7,3)$. Plot these points. What are the lengths of the parallel sides of the trapezoid?
10. Draw a sketch to show the addition on a number line of ${}^-3$ and ${}^-5$.
11. What is the coordinate of the point on the number line midway between the points given?
    (a) (4) and $({}^-6)$ (b) $({}^-10)$ and $({}^-1)$

Answers to Sample Questions.

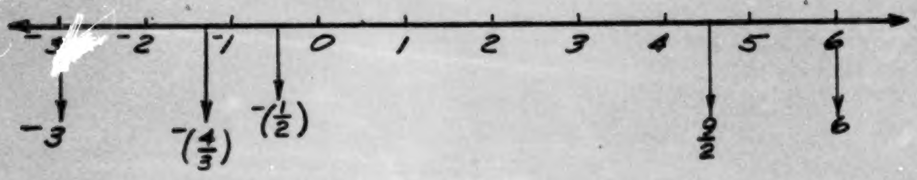
1. (a) $m - \ell = k$

   (b) $k = \frac{n}{\ell}$

2. (a) 1 (c) 17 (e) 7

   (b) ⁻4 (d) 2

3.


⁻3
⁻2
⁻1
0
1
2
3
4
5
6
⁻3
⁻(4/3)
⁻(1/2)
9/2
6

4. $\frac{(^-8) + (^-3) + (^-1) + 0 + 10}{5} = {}^-(\frac{2}{5})$

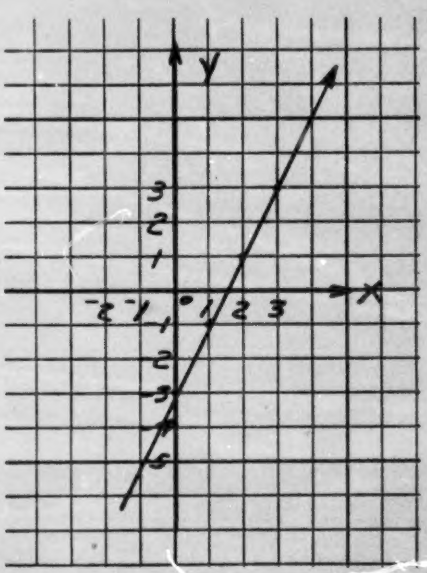
5.

x	⁻2	⁻1	0	1	4
y	6	3	0	⁻3	⁻12

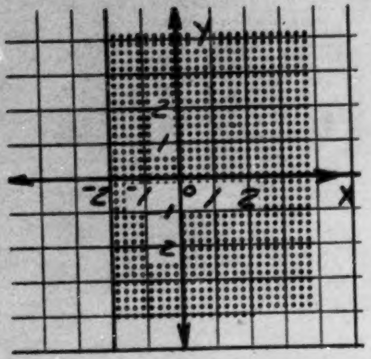
6.

x	⁻1	0	1	2	3
y	⁻5	⁻3	⁻1	1	3

6. Graph of $y = 2x - 3$



7. Graph of $x > {}^{-}2$

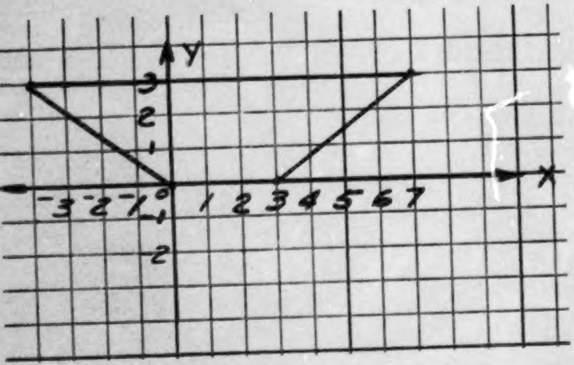


8. 48

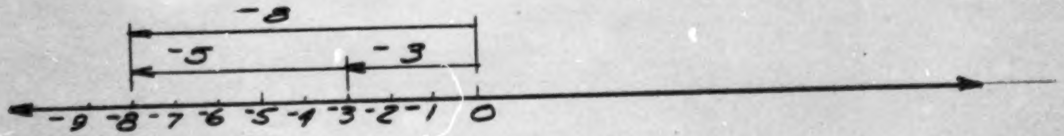
9.

(a) longest side is 11

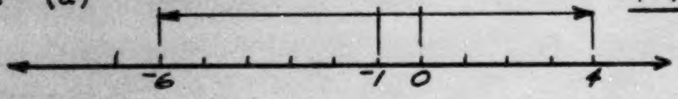
(b) shortest side is 3



10.

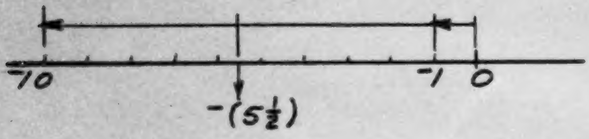

⁻8
⁻5
⁻3
⁻9
⁻8
⁻7
⁻6
⁻5
⁻4
⁻3
⁻2
⁻1
0

11. (a)



$$\frac{4 + (^-6)}{2} = \frac{^-2}{2} = {}^-1$$

(b)



$^-10$
$^-1$
0
$^-(5\frac{1}{2})$

$$\frac{(^-10) + (^-1)}{2} = (\frac{11}{2}) = {}^-(5\frac{1}{2})$$

Chapter 2

EQUATIONS

Introduction.

It is expected that 15 days will be required for this chapter. This should allow sufficient time for covering the material included in this chapter, a day for testing, and a day for discussion and review following the test.

Tell the pupils that they cannot read mathematics like a novel. They should have pencil and paper handy while reading the text, and they should work out the answers to any questions in a paragraph before going on to the next. They cannot absorb ideas like sponges. They must take an active part in the learning process. Although the authors of this chapter have tried to keep the reading difficulty at the junior high school level, this is not always possible due to the material covered. Careful reading on the part of the pupil, coupled with participation in carefully prepared class discussions should lead the pupil to good understanding of the ideas covered in this chapter.

At this time, we are trying to convey the meanings of concepts related to solving equations and to give the pupils some of the technical vocabulary he will need later. We are not trying, in this chapter, to develop great skill in solving equations, nor do we expect complete mastery of the subject. These goals are left to the formal, systematic instruction in algebra in the 9th grade. Hence we have avoided introducing such terms as "variable" and "constant." We aimed at the introduction of these ideas but felt it desirable to delay the introduction of the vocabuary until a more formal approach to equations is used.

It is advisable, therefore, to avoid dwelling at great length on this chapter. Pupils will have ample practice with equations later. Do not expect the pupils to achieve complete mastery of topics normally covered in the 9th grade.

2-1. Finding the Unknown.

Page 59. This section is intended to interest pupils in the solution of equations. The objective of this section is to explain what solving equations means and why it is important. Do not spend too much time on this section. We are not aiming at skill or mastery at this stage.

Some general review of work with formulas is presented in the exercises. The purpose here, in addition to the review, is to help the student see how a formula is related to an equation. Some teachers may also introduce a little practice in translating simple, real-life situations into the forms of equations. However, not much of this should be done at this time. When it is done, here and later in the chapter, it is important to write explicitly on the blackboard such statements as:

Let x = the number of years in Ann's age.

Emphasize to the class that when you formulate or solve a problem, the thinking is presented in an orderly manner. Thus, anyone who comes into the room should be able to understand the work on the board. Your chalk-board work should be a model for the way the children write their homework problems. The solving of equations in this section is to be done intuitively. Do not attempt to introduce any particular properties at this time. It may be necessary to review the customary way of writing the product of an unknown and a counting number as $3x$ instead of $3 \cdot x$ or $(3) \cdot (x)$, etc. It was assumed that the pupils would be familiar with the notations used.

Page 61. The references to conferences at the National Bureau of Standards are:

U.S. National Bureau of Standards, Applied Mathematics Series:
No. 29(1953). Simultaneous linear equations and the determination of eigenvalues. Edited by L.J. Paige and Olga Taussky.
No. 39(1954). Contributions to the solution of systems of linear equations and the determination of eigenvalues. Edited by Olga Taussky.
U.S. Government Printing Office, Washington, D.C.

Answers to Exercises 2-1. Page 61.

1. (a) $x = 2$
   (b) $y = 7$
   (c) $k = 2$
   (d) $m = 6$
   (e) $s = 25$
   (f) $t = {}^{-}5$

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2. (a) $x = 9$
   (b) $y = 10$
   (c) $n = 11$
   (d) $x = 9$
   (e) $p = 14$
   (f) $x = 8$

3. (a) $b = 3$
   (b) $a = 3$
   (c) $w = 7$
   (d) $m = 8$
   (e) $x = {}^{-}1$
   (f) $y = {}^{-}8$

4. (a) $n = 6$
   (b) $a = 16$
   (c) $k = {}^{-}16$
   (d) $d = 18$
   (e) $n = {}^{-}15$
   (f) $s = 21$

5. $p = 22$ (feet)

6. $A = 49$ (square inches)

Page 63.

7. $A = 225$ (square inches)

8. $i = 135$ (dollars)

9. $c = 62\frac{6}{7}$ (inches) or 62.8 (inches)

10. $d = 585$ (miles)

11. $p = 142.50$ (dollars)

12. $A = 531\frac{1}{7}$ (square feet) or 531.66 (square feet)

Page 64.

13. $A = 81$ (square feet)

*14. (a) $v = 11$ (cubic feet)
     (b) Capacity is $82\frac{1}{2}$ gallons.

*15. (a) 32° F
     (b) 212° F
     (c) 98.6° F

2-2. Number Phrases.

Page 65. The objective of this section is to teach the meaning of algebraic expressions, or "number phrases." The term, number phrase, is used here to agree with terminology used later on. The teacher may use additional examples to insure that the pupils understand that a phrase represents a number. When a letter is used in a phrase so that the phrase represents any one, or several, or all of a set of numbers the term "open phrase" is used. Whenever the value of the letter is chosen, an open phrase becomes a definite number.

Page 66. The class should be given sufficient practice in translating number phrases given in words into phrases in symbolic language, and in translating from symbols into words. Great care must be used with the terms "difference" and "quotient". Since the commutative property does not hold for the operations of subtraction and division, the expression, "the difference of 7 and 8" may not have any meaning. Should this be interpreted as 8 - 7 or 7 - 8? Having been introduced to negative numbers in Chapter 1, the pupil should learn to be very careful about expressions which may not have been questioned previously but which now appear ambiguous.

In the exercises the pupil is sometimes asked to find the value of an algebraic expression using a given value of the unknown. It might be pointed out to the pupil that the number represented by the phrase may be different if a different value for the unknown is used. This should help the pupil understand the idea of "open" phrases.

Answers to Exercises 2-2. Page 66.

1. (a) $x + 5$
   (b) $x - 3$
   (c) $8x$
   (d) $\frac{1}{4}x$ or $\frac{x}{4}$
   (e) $x + 10$
   (f) $7x$
   (g) $x - 11$
   (h) $\frac{x}{2}$
   (i) $x - 6$
   (j) $x - 9$

Page 67.

2. (a) 17
(b) 9
(c) 96
(d) 3
(e) 22
(e) 84
(f) 1
(g) 6
(h) 6
(i) 3

3. Pupils will undoubtedly write different translations for each of these phrases. Teachers should be primarily concerned with correct order of numbers when different versions are presented by pupils.

(a) The number x increased by 1.
(b) The number x decreased by 3.
(c) The number 2 multiplied by x.
(d) The number 18 divided by x.
(e) The number 4 multiplied by x.
(f) The number $^{-}6$ added to x.

4. (a) 7
(b) 3
(c) 12
(d) 3
(e) 24
(f) 0

5. (a) $^{-}1$
(b) $^{-}5$
(c) $^{-}4$
(d) $^{-}9$
(e) $^{-}8$
(f) $^{-}8$

6. (a) $6 + a$
(b) $8b$
(c) $8c + 1$
(d) $8d - 3$
(e) $\frac{8e}{4}$
(f) $2f + 3$
(g) $5(g + 2)$
(h) $7h - 10$
(i) $\frac{12}{i + 1}$
(j) $(j + 3)(j + 4)$

Page 68.

7. (a) 3
(b) $^{-}24$
(c) $^{-}23$
(d) $^{-}27$
(e) $^{-}6$
(f) $^{-}3$
(g) $^{-}5$
(h) $^{-}31$
(i) $^{-}6$
(j) 0

8. (a) Five more than two times a certain number.
   (b) Three times a certain number and the result subtracted from six.
   (c) A number decreased by one and the result multiplied by 7.
   (d) Five decreased by a number and the quantity divided by two.
   (e) Fifteen increased by two times a number.

9. (a) 15
   (b) $^{-}5$
   (c) 5
   (d) 3

10. (a) 6
    (b) 9
    (c) 3
    (d) $^{-}9$

2-3. Number Sentences.

Page 69. The objective of this section, as well as the preceding one, is to build the pupil's mathematical vocabulary. He should understand the terms:

number phrase, open phrase, number sentence, open sentence,
equation, inequality, solution set (or set of solution).

He should also be able to translate simple situations into mathematical language.

Page 70. Notice that a sentence expresses a propostion, something proposed for consideration. It may be true, or it may be false. You may want to introduce examples such as

$$x = x + 3$$

or

$$0 \cdot x = 1$$

which are false no matter what number x is.

Similarly, the number sentences

$$x + 5 = 5 + x$$

or

$$7x = x \cdot (7)$$

are true no matter what number x is.

Page 73. The material on representations on the number line is necessarily brief. However, though brief, having done this work the pupil should be able to understand better the ideas involved in sets of solutions. Relations of numbers should be much easier to understand when sets of solutions are viewed as a representation on the number line. These representations should be especially helpful in explaining sets of solutions of inequalities.

The term "graphing" was not introduced and used with the representations on the number line. We felt that to do so would needlessly complicate the work in later sections and would probably confuse the pupil when graphing of two unknowns is introduced.

Pages 76-79. The ideas involved in compound sentences is touched upon very briefly. The terms "conjunction" and "disjunction" are left to the 9th grade work.

The next part of this section helps summarize the material covered in the first part of the chapter. We tried here to point out to the pupil the advantages in translating a problem situation into symbolic form. The teacher may find it necessary to emphasize some of the main points of this section. For example, the pupil should be taught to make statements such as,

Let x = the number of years in John's age.

before writing a number sentence. It was assumed in this section that the pupils are familiar with the symbols "<" and ">".

Answers for Exercises 2-3a. Page 72.

1. (a) $x + 5 = 13$
   (b) $x - 3 = 7$
   (c) $8x = 24$
   (d) $\frac{x}{4} = 9$
   (e) $x + 10 = 21$
   (f) $7x = {}^{-}35$
   (g) $x - 11 = {}^{-}5$
   (h) $x - 6 = 15$
   (i) $\frac{x}{2} = {}^{-}7$

2. (a) $\{8\}$
   (b) $\{10\}$
   (c) $\{3\}$
   (d) $\{36\}$
   (e) $\{11\}$
   (f) $\{^-5\}$
   (g) $\{6\}$
   (h) $\{21\}$
   (i) $\{^-14\}$

3. (a) $x + 2 > 4$
   (b) $5x < 10$
   (c) $\frac{x}{7} > 2$
   (d) $x - 3 > 6$
   (e) $x - 5 < 13$
   (f) $3x > {}^-9$

Page 73.

4. (a) The set of all numbers greater than 2
   (b) The set of all numbers less than 2
   (c) The set of all numbers greater than 9
   (d) The set of all numbers greater than 14
   (e) The set of all numbers less than 18
   (f) The set of all numbers greater than $^-3$.

5. (a) Two more than a certain number is equal to five.
   (b) The sum of a certain number and negative three is seven.
   (c) Two multiplied by a number is equal to negative ten.
   (d) A number decreased by five is greater than nine.
   (e) The product of five and a number is less than fifteen.
   (f) Seven decreased by a number is two.
   (g) Three subtracted from a number is less than four.
   (h) A number divided by 3 is greater than nine.
   (i) A number decreased by seven is equal to negative two.
   (j) A number divided by $^-30$ is equal to six.

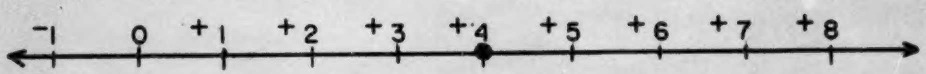
6. (a) $\{3\}$
   (b) $\{10\}$
   (c) $\{^-5\}$
   (d) The set of all numbers greater than 14
   (e) The set of all numbers less than 3
   (f) $\{5\}$

(g) The set of all numbers less than 7
(h) The set of all numbers greater than 27
(i) {5}
(j) $\{^{-}180\}$

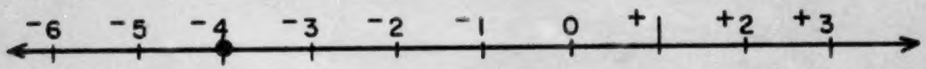
Answers for Exercises 2-3b. Page 76.

1. (a) {4}
   (b) $\{^{-}4\}$
   (c) {3}
   (d) {all numbers less than 1}
   (e) {all numbers greater than 5}
   (f) $\{^{-}5\}$
   (g) {all numbers less than 5}
   (h) {all numbers less than 3}

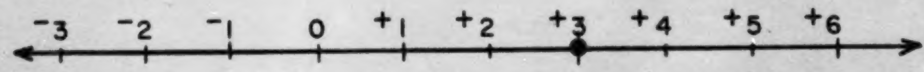
2. (a)


⁻1 0 ⁺1 ⁺2 ⁺3 ⁺4 ⁺5 ⁺6 ⁺7 ⁺8

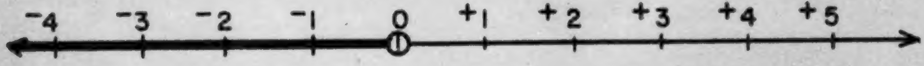
(b)


⁻6 ⁻5 ⁻4 ⁻3 ⁻2 ⁻1 0 ⁺1 ⁺2 ⁺3

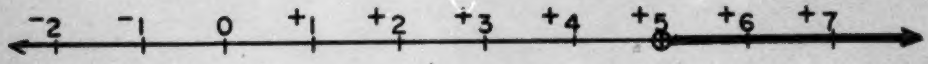
(c)


⁻3 ⁻2 ⁻1 0 ⁺1 ⁺2 ⁺3 ⁺4 ⁺5 ⁺6

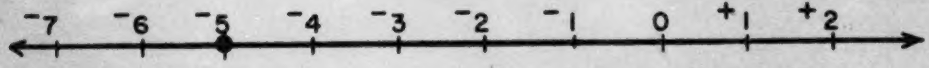
(d)


⁻4 ⁻3 ⁻2 ⁻1 0 ⁺1 ⁺2 ⁺3 ⁺4 ⁺5

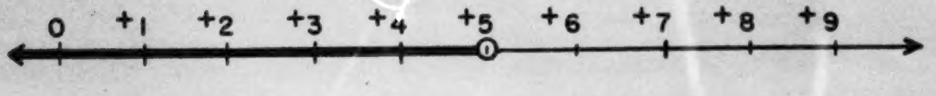
(e)


⁻2 ⁻1 0 ⁺1 ⁺2 ⁺3 ⁺4 ⁺5 ⁺6 ⁺7

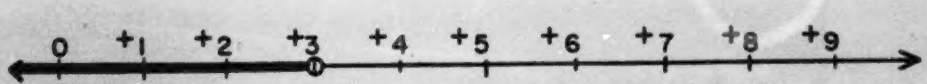
(f)


⁻7 ⁻6 ⁻5 ⁻4 ⁻3 ⁻2 ⁻1 0 ⁺1 ⁺2

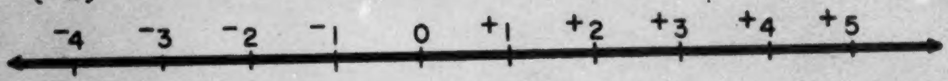
(g)

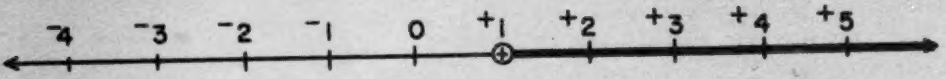

0 ⁺1 ⁺2 ⁺3 ⁺4 ⁺5 ⁺6 ⁺7 ⁺8 ⁺9

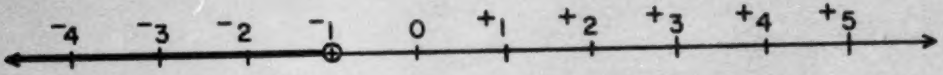
(h)

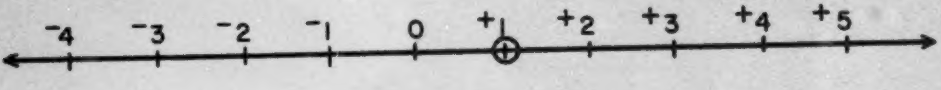

0 ⁺1 ⁺2 ⁺3 ⁺4 ⁺5 ⁺6 ⁺7 ⁺8 ⁺9

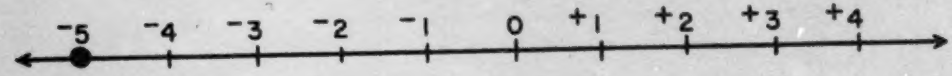
3. (a) {all numbers}
   (b) {all numbers greater than 1}
   (c) {all numbers less than 1}
   (d) The empty set. (The sentence has no solutions.)
   (e) $\{^-5\}$
   (f) $\{^-1\}$
   (g) $\{^-1\}$
   (h) $\{^-2\}$

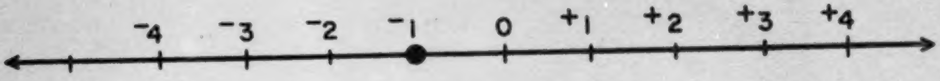
4. (a)

-4 -3 -2 -1 0 +1 +2 +3 +4 +5

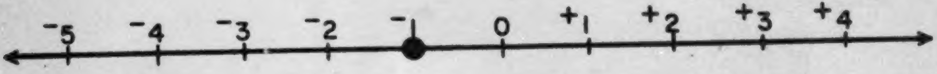
(b)

-4 -3 -2 -1 0 +1 +2 +3 +4 +5

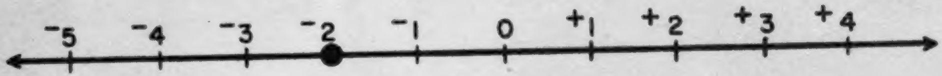
(c)

-4 -3 -2 -1 0 +1 +2 +3 +4 +5

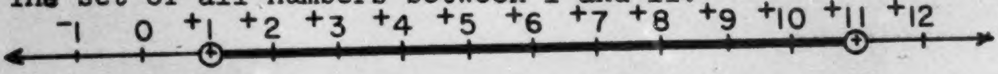
(d)

-4 -3 -2 -1 0 +1 +2 +3 +4 +5

(e)

-5 -4 -3 -2 -1 0 +1 +2 +3 +4

(f)


(g)

-5 -4 -3 -2 -1 0 +1 +2 +3 +4

(h)

-5 -4 -3 -2 -1 0 +1 +2 +3 +4

5. The set of all numbers between 1 and 11.

-1 0 +1 +2 +3 +4 +5 +6 +7 +8 +9 +10 +11 +12

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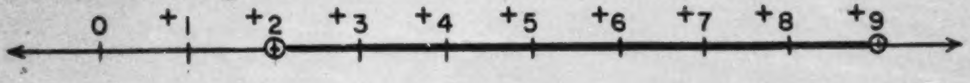
6. The intersection

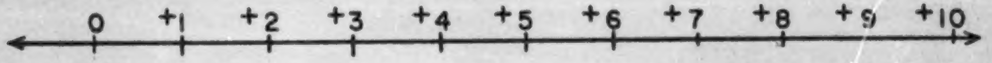
7. (a) The set of all numbers less than 9 and greater than 2.
   (b) The empty set.
   (c) All numbers greater than 3 and less than 6.

8. (a)



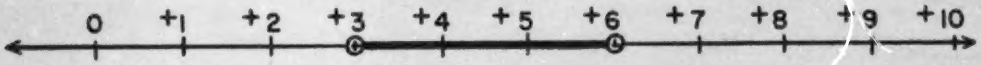
0
+1
+2
+3
+4
+5
+6
+7
+8
+9

(b)



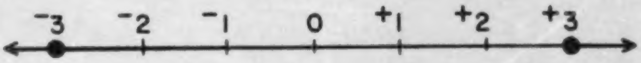
0
+1
+2
+3
+4
+5
+6
+7
+8
+9
+10

(c)



0
+1
+2
+3
+4
+5
+6
+7
+8
+9
+10

*9. (a) $\{3, {}^-3\}$
   (b)



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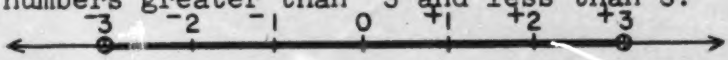
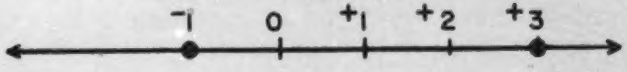
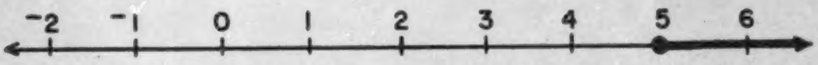
*10. (a) All numbers greater than $^-3$ and less than 3.
   (b)



*11. (a) For $x + 7 = 6$ $x = {}^-1$
   For $2x - 1 = 5$ $x = 3$
   The set of solutions is $\{{}^-1, 3\}$
   (b)



*12. (a) For $x - 1 = 4$ $x = 5$
   For $x - 1 > 4$ x is all numbers greater than 5.
   The set of solutions is 5 and all numbers greater than 5.
   (b)



*13. The solution set of "$x < 10$" is all numbers less than 10. The solution set of "$x - 9 > 0$" is all numbers greater than 9. Every number is in one of these two sets (or both). Thus the solution set of the compound sentence is the set of all numbers.

Answers To Exercises 2-3c. Page 81.

1. (a) y is the number of years in his sister's age.

   $10 = y - 4$

   (b) m is the number of model plane kits.

   $25m = 75$

   (c) b is the number of years in Sam's age now.

   $b + 7 = 20$

   (d) f is the number of feet.

   $12f = 72$

   (e) f is the number of feet.

   $\frac{f}{3} = 5$

   (f) n is the number of years in Ann's age now.

   $n - 10 = 3$

   (g) d is the number of dollars.

   $100d = 450$

   (h) x is the number of cents earned per hour before midnight.

   y is the number of cents earned per hour after midnight.

   $4x + 2y = 350$

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2. (a) d is the number of dollars that Dick has.

   $3 + 2x < 23$

   (b) r is the number of miles per hour that the plane travels.

   $2r > 500$

   (c) a is the number of years in Susie's age.

   $1 + 2a = 19$

   (d) x is the number of hours it takes to drive from Mill City to Dover.

   $240 = 40x$ or $\frac{240}{40} = x$

   (e) y is the number of cents that Sally will earn in 5 hours.

   $y = 5 \cdot 65$

3. (a) I a is the number of years in Mr Smith's age now.
II $a + 10 = 40$
III $a = 30$
(b) I x is the number of dollars you have now.
II $x + 5 = 12$
III $x = 7$
(c) I b is the number of inches in brother's height.
II $2b = 64$
III $b = 32$
(d) I p is the number of the year in which Paul was born.
II $p + 14 = 1958$
III $p = 1944$
(e) I h is the number of inches in the length of the altitude.
II $\frac{1}{2} \cdot 7h = 35$
III $h = 10$
(f) I y is the number.
II $\frac{20}{100}y = 10$
III $y = 50$
(g) I p is the number of dollars in the original price.
II $p - \frac{20}{100}p = 28$ or $\frac{80}{100}p = 28$
III $p = 35$
(h) I x is the number of miles in the complete journey.
II $120 + \frac{5}{8}x = x$
III $x = 320$

2-4. Finding Solution Sets

Page 84. In this section we study some of the properties of the relation of equality and their application to the solution of simple equations.

Pages 85-89. You should notice three things:

1. No subtraction property or division property is needed. It would only complicate things to bring in such properties. Since $a - c = a + {}^{-}c$ and $\frac{a}{c} = a \cdot (\frac{1}{c})$ these properties are implied by the addition and multiplication properties.

2. The addition and multiplication properties themselves are not "axioms," nor do they add anything new to our fund of information about numbers, equality, and the operations. For instance, we already know that addition is associative, that $c + {}^{-}c = 0$, that $b + 0 = b$, and that subtraction is the inverse operation for addition. Thus, if, $a = b$ then $a = b + 0$ or $a = b + (c + {}^{-}c)$. By the associative property, $a = (b + c) + {}^{-}c$ or $a = (b + c) - c$. By the very definition of subtraction this last statement is equivalent to $a + c = b + c$, and we have proved what is called the "addition property" in the text. You may wish to point this out to your class.

3. We could perfectly well solve the simple equations in this section without any mention of the addition and multiplication properties. For example, $z + 3 = {}^{-}7$ is equivalent to $z = {}^{-}7 - 3 = {}^{-}10$ by the definition of subtraction. We have chosen to present the material as it is in the text for two reasons: (a) These properties are important and useful in themselves; working with them gives excellent opportunity for review of the associative, commutative, and even the distributive properties of the operations. (b) We wish to avoid planting the seed of "transfer-from-one-side-to-the-other-and-change-the-sign."

The addition and multiplication properties show the relation between the equality relation and the arithmetic operations. The "<" has similar properties:

If x, y, and z are numbers and $x < y$, then $x + z < y + z$.

If x, y, and z are numbers and $x < y$ and $z > 0$, then $x \cdot z < y \cdot z$. Notice that this property differs slightly from the multiplication property of "=". If $z < 0$, then the last inequality would be reversed.

We apply the properties of "equals" to the solution of equations. Notice carefully the logic of the process. The usual reasoning in solving an equation actually proves a statement of the form:

> If x satisfies this equation, then $x =$ this or $x =$ that, etc. The logical function of checking is to prove the converse:

If $x =$ this or $x =$ that, etc., then x satisfies this equation. Thus the process of checking is an essential step in solving equations. Teach the pupils that their solution is not complete without the checking.

Answers to Exercises 2-4a. Page 87.

1. (a) (I) $x + 5 = 6$
   $(x + 5) + {}^-5 = 6 + {}^-5$ addition property, adding ${}^-5$
   $x + (5 + {}^-5) = 1$ associative property
   $x = 1$ $5 + {}^-5 = 0$
   (II) If $x = 1$ then $x + 5 = 1 + 5 = 6$, so 1 is a solution.

   (b) (I) $x + 6 = 5$
   $(x + 6) + {}^-6 = 5 + {}^-6$ addition property, adding ${}^-6$.
   $x + (6 + {}^-6) = {}^-1$ associative property
   $x = {}^-1$ $6 + {}^-6 = 0$
   (II) If $x = {}^-1$ then $x + 6 = {}^-1 + 6 = 5$, so ${}^-1$ is a solution.

   (c) (I) $x + {}^-7 = 7$
   $(x + {}^-7) + 7 = 7 + 7$ addition property, adding 7.
   $x + ({}^-7 + 7) = 14$ associative property
   $x = 14$ ${}^-7 + 7 = 0$
   (II) If $x = 14$, then $x + {}^-7 = 14 + {}^-7 = 7$, so 14 is a solution.

(d) (I) $x - 7 = {}^-7$

$x + {}^-7 = {}^-7$ property of additive inverse

$(x + {}^-7) + 7 = {}^-7 + 7$ addition property, adding 7

$x + ({}^-7 + 7) = 0$ associative property

$x = 0$ ${}^-7 + 7 = 0$

(II) If $x = 0$, then $x - 7 = 0 - 7 = 0 + {}^-7 = {}^-7$, so 0 is a solution.

(e) (I) $t + 6 = {}^-13$

$(t + 6) + {}^-6 = {}^-13 + {}^-6$ addition property, adding ${}^-6$

$t + (6 + {}^-6) = {}^-19$ associative property

$t = {}^-19$ $6 + {}^-6 = 0$

(II) If $t = {}^-19$, then $t + 6 = {}^-19 + 6 = {}^-13$, so ${}^-19$ is a solution.

(f) (I) $4 = s + 3$

$4 + {}^-3 = (s + 3) + {}^-3$ addition property, adding ${}^-3$

$1 = s + (3 + {}^-3)$ associative property

$1 = s$ $3 + {}^-3 = 0$

(II) If $s = 1$, then $s + 3 = 1 + 3 = 4$, or $4 = s + 3$, so 1 is a solution.

(g) (I) ${}^-2 = {}^-4 + x$

$4 + {}^-2 = 4 + ({}^-4 + x)$ addition property, adding 4

$2 = (4 + {}^-4) + x$ associative property

$2 = x$ $4 + {}^-4 = 0$

(II) If $x = 2$, then ${}^-4 + x = {}^-4 + 2 = {}^-2$, or ${}^-2 = {}^-4 + x$, so 2 is a solution.

(h) (I) $x + \frac{3}{2} = 10$

$(x + \frac{3}{2}) + {}^-(\frac{3}{2}) = 10 + {}^-(\frac{3}{2})$ addition property, adding ${}^-(\frac{3}{2})$

$x + (\frac{3}{2} + {}^-(\frac{3}{2})) = \frac{17}{2}$ associative property

$x = \frac{17}{2}$ $\frac{3}{2} + {}^-(\frac{3}{2}) = 0.$

(II) If $x = \frac{17}{2}$, then $x + \frac{3}{2} = \frac{17}{2} + \frac{3}{2} = \frac{20}{2} = 10$, so $\frac{17}{2}$ is a solution.

(i) (I) $y - \frac{3}{2} = \frac{5}{2}$

$y + {}^{-}(\frac{3}{2}) = \frac{5}{2}$

$(y + {}^{-}(\frac{3}{2})) + \frac{3}{2} = \frac{5}{2} + \frac{3}{2}$ addition property, adding $\frac{3}{2}$

$y + ({}^{-}(\frac{3}{2}) + \frac{3}{2}) = \frac{8}{2} = 4$ associative property

$y = 4$ ${}^{-}(\frac{3}{2}) + \frac{3}{2} = 0$

(II) If $y = 4$, then $y - \frac{3}{2} = 4 - \frac{3}{2} = \frac{8}{2} - \frac{3}{2} = \frac{5}{2}$, or $y - \frac{3}{2} = \frac{5}{2}$, so 4 is a solution.

(j) (I) $u + 14 = \frac{9}{5}$

$(u + 14) + {}^{-}14 = \frac{9}{5} + {}^{-}14$ addition property, adding ${}^{-}14$

$u + (14 + {}^{-}14) = \frac{9}{5} + {}^{-}(\frac{70}{5}) = {}^{-}(\frac{61}{5})$ associative property

$u = {}^{-}(\frac{61}{5})$ $14 + {}^{-}14 = 0$

(II) If $u = {}^{-}(\frac{61}{5})$, then $u + 14 = {}^{-}(\frac{61}{5}) + 14 = {}^{-}(\frac{61}{5}) + \frac{70}{5} = \frac{9}{5}$, or $u + 14 = \frac{9}{5}$, so ${}^{-}(\frac{61}{5})$ is a solution.

(k) (I) $\frac{13}{7} = 1 + x$

${}^{-}1 + \frac{13}{7} = {}^{-}1 + (1 + x)$ addition property, adding ${}^{-}1$

$\frac{6}{7} = ({}^{-}1 + 1) + x$ associative property

$\frac{6}{7} = x$ ${}^{-}1 + 1 = 0$

(II) If $x = \frac{6}{7}$, then $1 + x = 1 + \frac{6}{7} = \frac{7}{7} + \frac{6}{7} = \frac{13}{7}$, or $\frac{13}{7} = 1 + x$, so $\frac{6}{7}$ is a solution.

(l) (I) $x + {}^{-}(\frac{4}{9}) = {}^{-}(\frac{7}{13})$

$(x + {}^{-}(\frac{4}{9})) + \frac{4}{9} = {}^{-}(\frac{7}{13}) + \frac{4}{9}$ addition property, adding $\frac{4}{9}$

$x + ({}^{-}(\frac{4}{9}) + \frac{4}{9}) = {}^{-}(\frac{63}{117}) + \frac{52}{117} = {}^{-}(\frac{11}{117})$ associative property

$x = {}^{-}(\frac{11}{117})$ ${}^{-}(\frac{4}{9}) + \frac{4}{9} = 0$

(II) If $x = {}^{-}(\frac{11}{117})$, then $x + {}^{-}(\frac{4}{9}) = {}^{-}(\frac{11}{117}) + {}^{-}(\frac{4}{9}) =$ ${}^{-}(\frac{11}{117}) + {}^{-}(\frac{4}{9}) = {}^{-}(\frac{11}{117}) + {}^{-}(\frac{52}{117}) = {}^{-}(\frac{63}{117})$ and ${}^{-}(\frac{63}{117}) =$ ${}^{-}(\frac{7\cdot 9}{13\cdot 9}) = {}^{-}(\frac{7}{13})$. Thus $x + {}^{-}(\frac{4}{9}) = {}^{-}(\frac{7}{13})$, and ${}^{-}(\frac{11}{117})$ is a solution.

2. (a) $2x + 5 = 10$

$(2x + 5) + {}^-5 = 10 + {}^-5$ addition property

$2x + (5 + {}^-5) = 5$ associative property

$2x = 5$

(b) $3x + 10 = 5$

$(3x + 10) + {}^-10 = 5 + {}^-10$ addition property

$3x + (10 + {}^-10) = {}^-5$ associative property

$3x = {}^-5$

(c) $5x + 2 = {}^-2$

$(5x + 2) + {}^-2 = {}^-2 + {}^-2$ addition property

$5x + (2 + {}^-2) = {}^-4$ associative property

$5x = {}^-4$

(d) $10x + {}^-1 = 9$

$(10x + {}^-1) + 1 = 9 + 1$ addition property

$10x + ({}^-1 + 1) = 10$ associative property

$10x = 10$

(e) $2u + 1 = 11$

$(2u + 1) + {}^-1 = 11 + {}^-1$ addition property

$2u + (1 + {}^-1) = 10$ associative property

$2u = 10$

(f) $2x + {}^-3 = 9$

$(2x + {}^-3) + 3 = 9 + 3$ addition property

$2x + ({}^-3 + 3) = 12$ associative property

$2x = 12$

(g) ${}^-4y + {}^-3 = \frac{9}{2}$

$({}^-4y + {}^-3) + 3 = \frac{9}{2} + 3 = \frac{9}{2} + \frac{6}{2} = \frac{15}{2}$ addition property

${}^-4y + ({}^-3 + 3) = \frac{15}{2}$ associative property

${}^-4y = \frac{15}{2}$

3. (a) ${}^-3$

(b) 7

(c) 3 and ${}^-3$ are additive inverses of each other (or inverses under addition).

(d) 7 and ${}^{-}7$ are additive inverses of each other (or inverses under addition).

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4. (a) $x + 3 = 11$

$(x + 3) + {}^{-}3 = 11 + {}^{-}3$ addition property

$x + (3 + {}^{-}3) = 8$ associative property

$x = 8.$

If $x = 8$, then $x + 3 = 8 + 3 = 11$, so 8 is a solution.

(b) $x + {}^{-}6 = \frac{4}{3}$

$(x + {}^{-}6) + 6 = \frac{4}{3} + 6 = \frac{4}{3} + \frac{18}{3}$ addition property

$x + ({}^{-}6 + 6) = \frac{22}{3}$ associative property

$x = \frac{22}{3}.$

If $x = \frac{22}{3}$, then $x + {}^{-}6 = \frac{22}{3} + {}^{-}6 = \frac{22}{3} + {}^{-}(\frac{18}{3}) = \frac{4}{3}$, so $\frac{22}{3}$ is a solution.

(c) $x - \frac{9}{2} = \frac{5}{2}$

$x + {}^{-}(\frac{9}{2}) = \frac{5}{2}$

$(x + {}^{-}(\frac{9}{2})) + \frac{9}{2} = \frac{5}{2} + \frac{9}{2}$

$x + ({}^{-}(\frac{9}{2}) + \frac{9}{2}) = \frac{14}{2} = 7$ associative property

$x = 7.$

If $x = 7$, then $x - \frac{9}{2} = 7 - \frac{9}{2} = \frac{14}{2} - \frac{9}{2} = \frac{5}{2}$, so 7 is a solution.

(d) $2x - 7 = x$

${}^{-}x + (2x - 7) = x + {}^{-}x$ addition property

$({}^{-}x + 2x) - 7 = 0$ associative property

$({}^{-}1 + 2)x - 7 = 0$ distributive property!

$x - 7 = 0$

$(x + {}^{-}7) + 7 = 0 + 7$ addition property

$x + ({}^{-}7 + 7) = 7$ associative property

$x = 7.$

If $x = 7$, then $2x - 7 = 14 - 7 = 7 = x$ so 7 is a solution.

(e) $^-x = 7 - 2x = 7 + {^-2x}$

$^-x + 2x = (7 + {^-2x}) + 2x$ addition property

$^-x + 2x = 7 + ({^-2x} + 2x)$ associative property

$({^-1} + 2)x = 7 + ({^-2} + 2)x$ distributive property

$x = 7.$

If $x = 7$, then $7 - 2x = 7 - 14 = {^-7} = {^-x}$, or ${^-x} = 7 - 2x$, so 7 is a solution.

(f) $\frac{1}{2}x + 2 = 1.5 - \frac{1}{2}x = 1.5 + {^-(\frac{1}{2})}x$

$\frac{1}{2}x + (\frac{1}{2}x + 2) = \frac{1}{2}x + ({^-(\frac{1}{2}x)} + 1.5)$ addition property and commutative property of addition.

$(\frac{1}{2}x + \frac{1}{2}x) + 2 = (\frac{1}{2}x + {^-(\frac{1}{2})x}) + 1.5$ associative property

$(\frac{1}{2} + \frac{1}{2})x + 2 = (\frac{1}{2} + {^-(\frac{1}{2})})x + 1.5$ distributive property

$x + 2 = 1.5$

$(x + 2) + {^-2} = 1.5 + {^-2}$ addition property

$x + (2 + {^-2}) = {^-.5}$ associative property

$x = {^-.5}$

If $x = {^-.5}$, then $\frac{1}{2}x + 2 = \frac{1}{2}({^-.5}) + 2 = {^-.25} + 2 = 1.75$, and $1.5 - \frac{1}{2}x = 1.5 - \frac{1}{2}({^-.5}) = 1.5 + {^-(\frac{1}{2})}({^-.5}) = 1.5 + 2.5 = 1.75$. Thus $\frac{1}{2}x + 2 = 1.75 = 1.5 - \frac{1}{2}x$, and $x = {^-.5}$ is a solution.

Answers to Class Exercises 2-4. Page 91.

1. (a) Addition
   (b) Addition
   (c) Addition
   (d) Multiplication
   (e) Multiplication
   (f) Addition (or Addition (${^-14}$) and Multiplication (${^-1}$)).
   (g) Multiplication
   (h) Addition
   (i) Addition
   (j) Multiplication
   (k) Addition (or Addition (${^-6}$) and Multiplication (${^-1}$)).

(l) Multiplication
(m) Multiplication
(n) Multiplication (by c).

2. (a) Multiplication property, multiplying by $\frac{1}{2}$.
(b) Multiplication property, multiplying by $\frac{1}{4}$.
(c) Multiplication property, multiplying by 3.
(d) Multiplication property, multiplying by $^{-}1$.
(e) Addition property, adding $^{-}1$.
(f) Multiplication property, multiplying by $\frac{1}{2}$.
(g) Multiplication property, multiplying by 10.
(h) Multiplication property, multiplying by n.
(i) Addition property, adding $^{-}3x = {}^{-}(3x)$.

3. (a) $7 + {}^{-}1 = (3x + 1) + {}^{-}1$
$6 = 3x + (1 + {}^{-}1)$
$6 = 3x.$

(b) $\frac{1}{3} \cdot 6 = \frac{1}{3}(3w)$
$2 = (\frac{1}{3} \cdot 3)w$
$2 = w.$

(c) $(\frac{t}{2} - 1.7) \times 2 = ({}^{-}1.3) \times 2$
$\frac{t}{2} \times 2 - (1.7) \times 2 = {}^{-}2.6;$ Distributive property
$t - 3.4 = {}^{-}2.6.$

(d) $b = \frac{x}{18}$
$18b = 18 \ (\frac{x}{18})$
$18b = (18 \cdot \frac{1}{18})x$
$18b = x.$

(e) ${}^{-}0.14 + (0.14 + x) = 5.28 + {}^{-}0.14$
$({}^{-}0.14 + 0.14) + x = {}^{-}0.14 + 5.28$
$x = 5.14$

(f) $5x - 7 = 2x$

$^{-}(2x) + (5x-7) = {}^{-}(2x) + 2x$

$(^{-}2x) + 5x) - 7 = 0$

$(^{-}2x + 5x) - 7 = 0$

$(-2 + 5)x - 7 = 0;$ distributive property

$3x - 7 = 0.$

(g) $(2x - 3y) + 3y = 4 + 3y$

$(2x + {}^{-}(3y)) + 3y = 4 + 3y$

$2x + ({}^{-}(3y) + 3y) = 4 + 3y$

$2x = 4 + 3y$

(h) $x + 2x = (7 - 2x) + 2x$

$(1 + 2)x = (7 + {}^{-}(2x)) + 2x$

$3x = 7 + ({}^{-}(2x) + 2x)$

$3x = 7$

4. Associative property of addition used in (a), (e), (f), (g), (h)
Associative property of multiplication used in (b), (d),
Distributive property used in (c), (f), (g), (h).

Answers to Exercises 2-4b. Page 92.

1. (a) $2x + 1 = 7$

$(2x + 1) + {}^{-}1 = 7 + {}^{-}1$ Addition property (Ad. P.)

$2x + (1 + {}^{-}1) = 6$ Associative property (As. P. A.)

$2x = 6$ $1 + {}^{-}1 = 0$

$\frac{1}{2}(2x) = \frac{1}{2} \cdot 6$ Multiplication property (M. P.)

$(\frac{1}{2} \cdot 2)x = 3$ Associative property of multiplication (As. P. M.)

$x = 3.$ $\frac{1}{2} \cdot 2 = 1$

If $x = 3$, then $2x + 1 = 2 \cdot 3 + 1 = 6 + 1 = 7$, so $2x + 1 = 7$. Thus 3 is a solution, the only solution. The solution set is $\{3\}$.

(b) $y - 2 = 6$; same as $y + {}^-2 = 6$

$(y + {}^-2) + 2 = 6 + 2$ Ad. P.

$y + ({}^-2 + 2) = 8$ As. P.A.

$y = 8.$ ${}^-2 + 2 = 0.$

If $y = 8$, then $y - 2 = 8 - 2 = 6$, so 8 is a solution. The solution set is $\{8\}$.

(c) $\frac{t}{2} - 3 = {}^-4$; same as $\frac{t}{2} + {}^-3 = {}^-4$

$(\frac{t}{2} + {}^-3) + 3 = {}^-4 + 3$ Ad. P.

$\frac{t}{2} + ({}^-3 + 3) = {}^-1$ As. P.A.

$\frac{t}{2} = {}^-1$ ${}^-3 + 3 = 0$

$(\frac{t}{2}) \cdot 2 = {}^-1 \cdot 2$ M. P.

$t(\frac{1}{2} \cdot 2) = {}^-2$ As. P. M.

$t = {}^-2.$ $\frac{1}{2} \cdot 2 = 1$

If $t = {}^-2$, then $\frac{t}{2} - 3 = (\frac{{}^-2}{2}) - 3 = {}^-1 - 3 = {}^-1 + {}^-3 = {}^-4$, so $\frac{t}{2} - 3 = {}^-4$, and ${}^-2$ is a solution. The solution set is $\{{}^-2\}$.

(d) $3x - 5 = {}^-4$; same as $3x + {}^-5 = {}^-4$

$(3x + {}^-5) + 5 = {}^-4 + 5$ Ad. P.

$3x + ({}^-5 + 5) = 1$ As. P. A.

$3x = 1$ ${}^-5 + 5 = 0.$

$\frac{1}{3}(3x) = \frac{1}{3} \cdot 1$ M. P.

$(\frac{1}{3} \cdot 3)x = \frac{1}{3}$ As. P. M.

$x = \frac{1}{3}.$ $\frac{1}{3} \cdot 3 = 1$

If $x = \frac{1}{3}$, then $3x - 5 = 3 \cdot \frac{1}{3} - 5 = 1 - 5 = {}^-4$, so $3x - 5 = {}^-4$. Thus $\frac{1}{3}$ is a solution. The solution set is $\{\frac{1}{3}\}$.

2. (a) $x + 3 = 5$

$(x + 3) + {}^-3 = 5 + {}^-3$

$x + (3 + {}^-3) = 2$

$x = 2.$

If $x = 2$, $x + 3 = 2 + 3 = 5$ so $x + 3 = 5$.

Solution set = $\{2\}$.

The remaining parts of Problem 2 should be solved by following the pattern of 2(a). Since the teacher is familiar with this pattern only the solution sets will be listed hereafter.

(b) The solution set is $\{^-8\}$.
(c) The solution set is $\{1\}$.
(d) The solution set is $\{^-4\}$.
(e) The solution set is $\{8\}$.
(f) The solution set is $\{8\}$.
(g) The solution set is $\{4\}$.
(h) The solution set is $\{4\}$.
(i) The solution set is $\{2\}$.
(j) The solution set is $\{^-2\}$.
(k) The solution set is $\{^-4\}$.
(l) The solution set is $\{2\}$.

3. (a) $\frac{1}{9}$ (d) They are inverses of each other, or reciprocals.
(b) 3 (e) They are inverses of each other, or reciprocals.
(c) $\frac{5}{4}$ (f) They are inverses of each other, or reciprocals.

Page 93.

4. (a) Addition Property: If a, b, and c are numbers, and $a < b$, then $a + c < b + c$ and $c + a < c + b$.
This is true.
Multiplication Property: If a, b, and c are numbers, and $a < b$, then $ac < bc$ and $ca < cb$. This is false!
Example: $2 < 4$, but $2 \cdot {}^-1 = {}^-2$, $4 \cdot {}^-1 = {}^-4$, and ${}^-2 < {}^-4$ is false!
The multiplication property for "<" is true if the condition, $c > 0$, is added to the statement.

(b) Addition Property: If a, b, and c are integers and a is a factor of b, then $a + c$ is a factor of $b + c$ and $c + a$ is a factor of $c + b$.
This is false! 2 is a factor of 4, but $2 + 1$ is not a factor of $4 + 1$.

Multiplication Property: If a, b, and c are integers and a is a factor of b then ac is a factor of bc and ca is a factor of cb.
This is true.

5. (a) $4(x + 1) = 12$
$\frac{1}{4}(4(x + 1)) = \frac{1}{4}\cdot 12$
$(\frac{1}{4}\cdot 4)(x + 1) = 3$
$x + 1 = 3$
$(x + 1) + {}^{-}1 = 3 + {}^{-}1$
$x + (1 + {}^{-}1) = 2$
$x = 2.$
If $x = 2$, then $4(x + 1) = 4(2 + 1) = 4\cdot 3 = 12$. Thus $4(x + 1) = 12$.
Solution set: $\{2\}$

(b) $7(x - 2) = 13$
$\frac{1}{7}(7(x - 2)) = \frac{1}{7}\cdot 13$
$(\frac{1}{7}\cdot 7)(x - 2) = \frac{13}{7}$
$x - 2 = \frac{13}{7}$
$(x + {}^{-}2) + 2 = \frac{13}{7} + 2$
$x + ({}^{-}2 + 2) = \frac{27}{7}$
$x = \frac{27}{7}$
If $x = \frac{27}{7}$, then $7(x - 2) = 7(\frac{27}{7} - 2) = 7\cdot(\frac{13}{7}) = (7\cdot\frac{1}{7})\cdot 13 = 13$, so $7(x - 2) = 13$.
Solution set: $\{\frac{27}{7}\}$

(c) $\frac{x + 9}{3} = 5$
$3\cdot(\frac{x + 9}{3}) = 3\cdot 5$
$(3\cdot\frac{1}{3})(x + 9) = 15$
$x + 9 = 15$
$(x + 9) + {}^{-}9 = 15 + {}^{-}9$
$x + (9 + {}^{-}9) = 6$
$x = 6$

If $x = 6$, then $\frac{x + 9}{3} = \frac{6 + 9}{3} = \frac{15}{3} = 5$.

Solution set: $\{6\}$

(d) $.6(x - .3) = .2$

$\frac{1}{.6} = \frac{1}{.6} \cdot \frac{10}{10} = \frac{10}{6}$ so,

$\frac{10}{6}(.6(x - .3)) = \frac{10}{6} \cdot .2$

$(\frac{10}{6} \cdot \frac{6}{10})(x - \frac{3}{10}) = \frac{10}{6} \cdot \frac{2}{10} = \frac{1}{3}$

$x - \frac{3}{10} = \frac{1}{3}$

$(x + {}^{-}(\frac{3}{10})) + \frac{3}{10} = \frac{1}{3} + \frac{3}{10}$

$x + ({}^{-}(\frac{3}{10}) + \frac{3}{10}) = \frac{10}{30} + \frac{9}{30}$

$x = \frac{19}{30}$.

If $x = \frac{19}{30}$, then $.6(x - .3) = \frac{6}{10}(\frac{19}{30} - \frac{3}{10}) = \frac{6}{10}(\frac{19-9}{30}) =$ $\frac{6}{10} \cdot (\frac{10}{30}) = \frac{6 \cdot 10}{30 \cdot 10} + \frac{6}{30} = \frac{2}{10} = .2$. Thus $.6(x - .3) = .2$

Solution set: $\{\frac{19}{30}\}$

(e) $\frac{3x + 4}{2} = 7$

$2(\frac{3x + 4}{2}) = 2 \cdot 7$

$(2 \cdot \frac{1}{2})(3x + 4) = 14$

$3x + 4 = 14$

$(3x + 4) + {}^{-}4 = 14 + {}^{-}4$

$3x + (4 + {}^{-}4) = 10$

$3x = 10$

$\frac{1}{3}(3x) = \frac{1}{3} \cdot 10$

$(\frac{1}{3} \cdot 3)x = \frac{10}{3}$

$x = \frac{10}{3}$

If $x = \frac{10}{3}$, $\frac{3x + 4}{2} = \frac{3 \cdot \frac{10}{3} + 4}{2} = \frac{10 + 4}{2} = \frac{14}{2} = 7$. Thus $\frac{3x + 4}{2} = 7$.

Solution set: $\{\frac{10}{3}\}$

(f) $\frac{4x + 1}{.12} = 3$

$.12\left(\frac{4x + 1}{.12}\right) = .12 \cdot 3$

$\left(.12 \cdot \frac{1}{.12}\right)(4x + 1) = .36$

$4x + 1 = .36$

$(4x + 1) + {}^{-}1 = .36 + {}^{-}1$

$4x + (1 + {}^{-}1) = {}^{-}.64$

$4x = {}^{-}.64$

$\frac{1}{4}(4x) = \frac{1}{4}({}^{-}.64)$

$\left(\frac{1}{4} \cdot 4\right)x = {}^{-}.16$

$x = {}^{-}.16$

If $x = {}^{-}.16$, $\frac{4x + 1}{.12} = \frac{4({}^{-}.16) + 1}{.12} = \frac{{}^{-}.64 + 1}{.12} = \frac{.36}{.12} =$
$\frac{.12 \cdot 3}{.12} = 3$. Thus $\frac{4x + 1}{.12} = 3$.

Solution set: $\{{}^{-}.16\}$

2-5. Number Sentences with Two Unknowns.

Page 94. This section introduces the important concept of an equation or inequality involving *two* unknowns. Here the solution is an ordered pair of numbers. This leads in turn to the representation of the solution of a number sentence by some type of graph.

It is especially important that the students work through this section with pencil and paper. They should all have graph paper before they begin reading this part. You may pattern the class discussion on the questions in the text.

Page 95. In studying ordered pairs of numbers, the teacher should emphasize the importance of *order* in performing operations, such as subtractions and divisions as well as in many non-mathematical situations.

In the class discussions of $y = x^2$, the students should notice that for each positive value of y there are two values for x, and that no value of x will give a negative value for y. It may be worthwhile making a magnified graph in the neighborhood of the origin taking x as 0, ± 0.1, ± 0.2, etc.

Page 93. In plotting straight lines it might be mentioned that locating two points is sufficient, but that as a check it is best to plot at least three points -- preferably more.

There is one exception to the general statement that the graph of $ax + by = c$ is a line. There is no need to discuss this topic in class, but the teacher should be prepared in case any bright student notices this possibility.

The exceptional case occurs when a and b are both zero. If c is also zero, the equation becomes $0x + 0y = 0$. This equation is satisfied by all values of x and y, and therefore the graph is the entire plane.

If however c is not zero, then the equation (for $c = 5$) becomes $0x + 0y = 5$. This equation is satified by no values of x and y, and the graph is the empty set.

The class will probably agree that whenever we refer to $ax + by = c$ as a linear equation, we are implying that at least one of the terms in x and y actually appears with a non-zero coefficient.

Answers to Questions in Section 2-5. Page 94.

1. The sentence: $x + 1 = y$
   If $x = 3$ and $y = 5$ the sentence is false.
   If $x = 7$, y must be 8.
   If $y = {}^-6$. x must be ${}^-7$.

   The table is:

x	y
0	1
1	2
2	3
${}^-(\frac{2}{3})$	$\frac{1}{3}$
${}^-(\frac{16}{3})$	${}^-(\frac{13}{3})$

Page 97.

2. The equation $2x + y = {}^{-}1$.

x	y
$^{-}3$	5
$^{-}1$	1
0	$^{-}1$
4	$^{-}9$
$\frac{1}{2}$	$^{-}2$
$^{-}(\frac{1}{2})$	0
$^{-}(\frac{3}{2})$	2

Page 98-99.

3. The number sentence "$2x + 2y = 16$ and $x > 0$, $y > 0$."

The pairs $(1,7)$, $(2,6)$, $(\frac{7}{2},\frac{9}{2})$, and $(5,3)$ are solutions.

The solutions $(1,7)$, $(2,6)$, and $(5,3)$ have been plotted on the graph .

Page 101.

4. The inequality $xy > 0$.

The blanks should contain positive , negative , first , third .

Page 102.

5. The equation $4y = x^2$.

x	y
$^{-}4$	4
$^{-}3$	$\frac{9}{4}$
$^{-}2$	1
$^{-}1$	$\frac{1}{4}$
0	0

x	y
1	$\frac{1}{4}$
2	1
3	$\frac{9}{4}$
4	4

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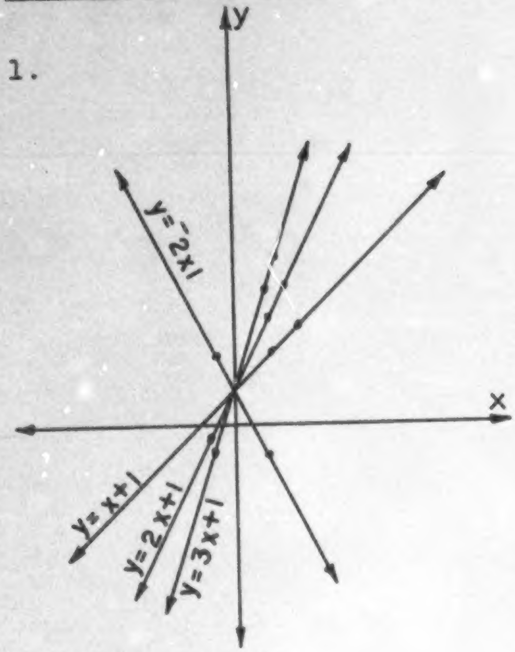
6. The equation $y = \frac{1}{4}x^2$.

If $y > \frac{1}{4}x^2$ the point (x,y) is above the parabola.

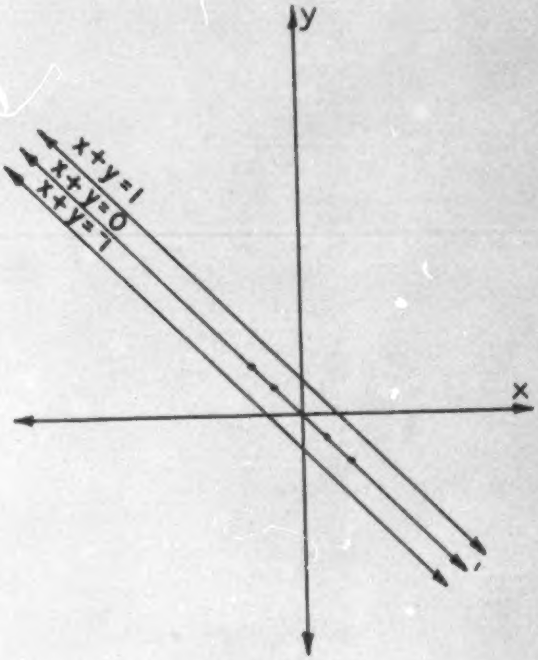
If (x,y) lies above the parabola then $y > \frac{1}{4}x^2$.

Answers to Exercises 2-5a. Page 98.

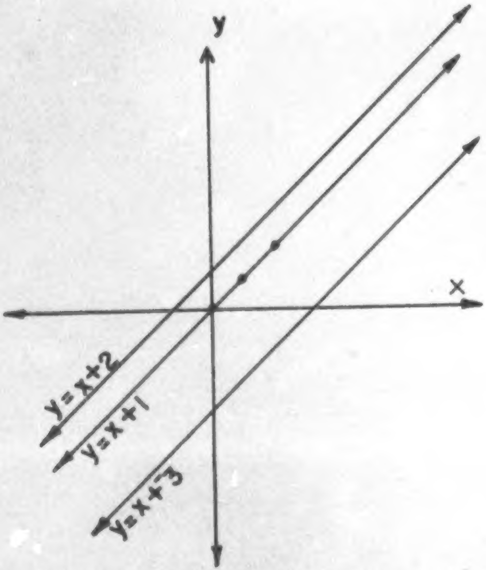
1.


y
x
y=⁻2x+1
y=x+1
y=2x+1
y=3x+1

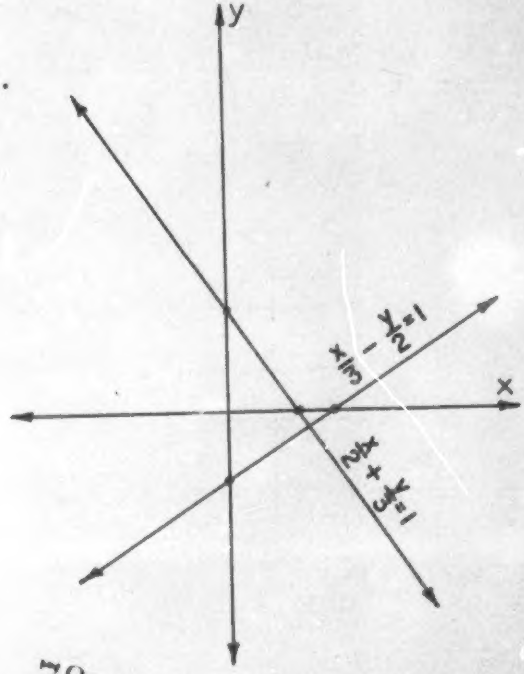
3.


y
x
x+y=1
x+y=0
x+y=⁻1

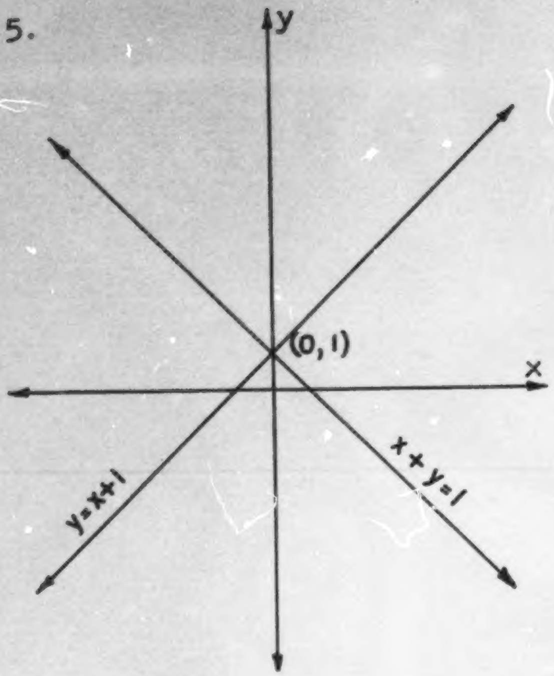
2.


y
x
y=x+2
y=x+1
y=x+3

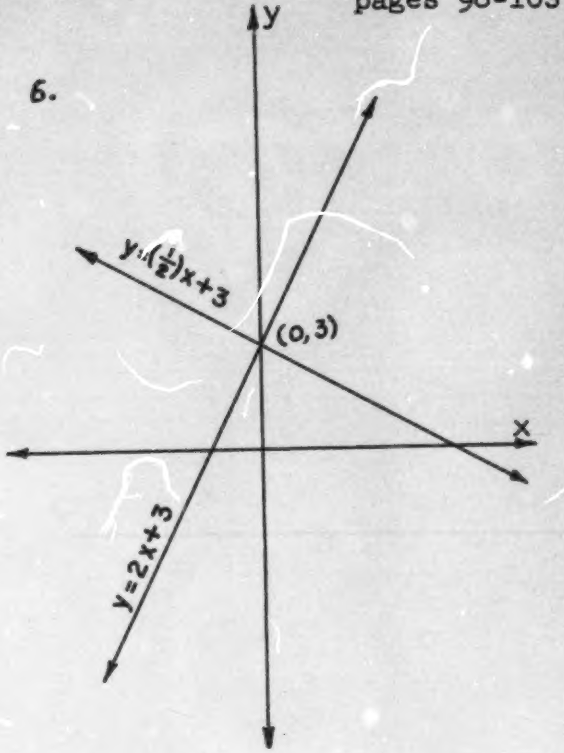
4.


y
x
$\frac{x}{3} - \frac{y}{2} = 1$
$\frac{x}{2} + \frac{y}{3} = 1$

5.

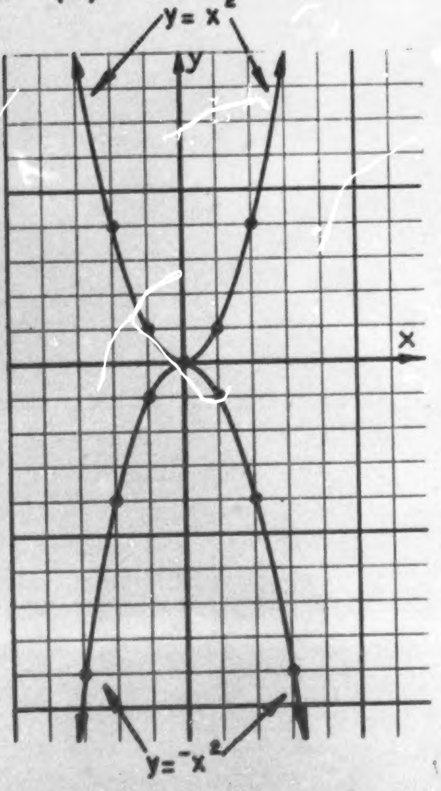


6.

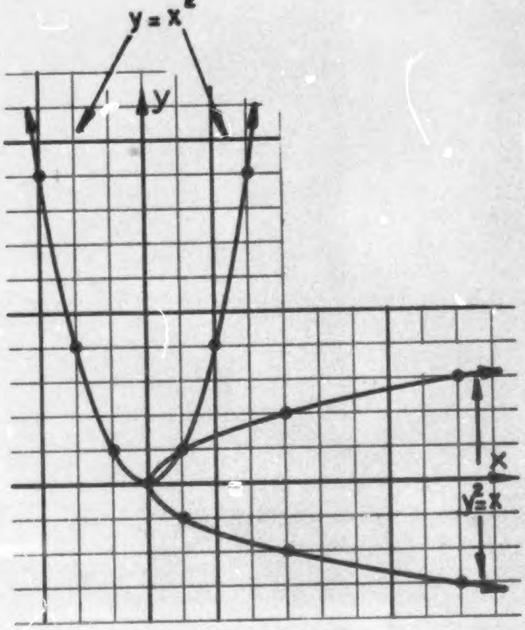


Answers to Exercises 2-5b. Page 103.

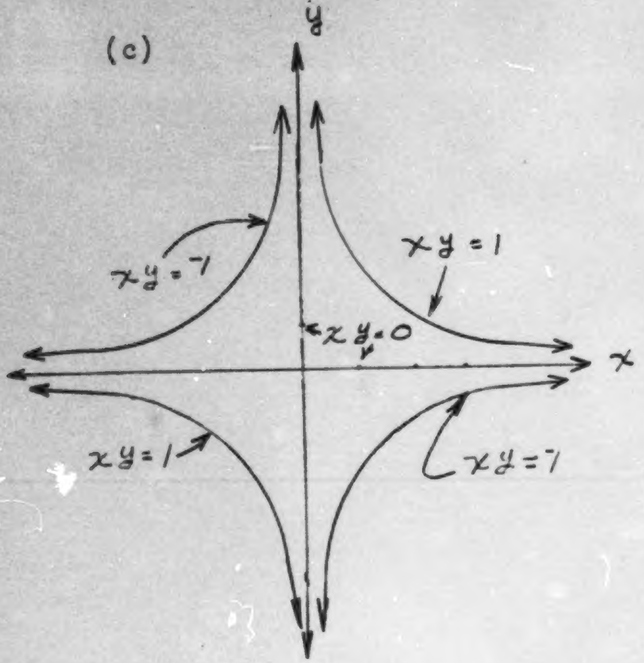
1. (a)



(b)



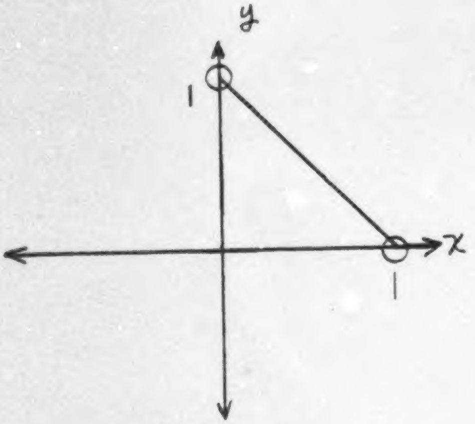
(c)


y
x
xy = -1
xy = 1
xy = 0
xy = 1
xy = -1

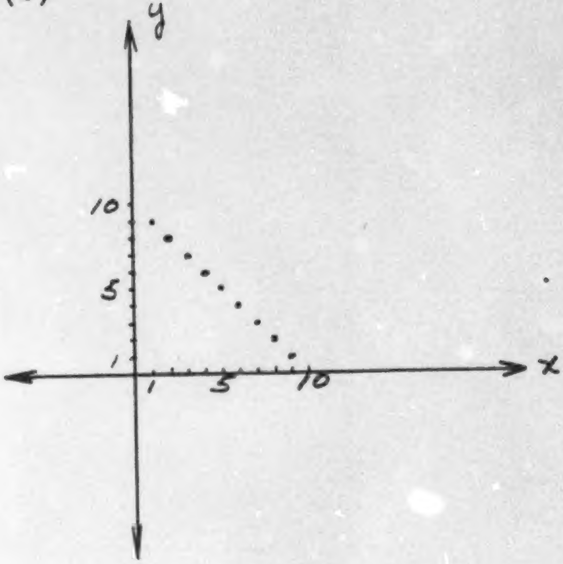
xy = 0 is the union of the set of points on the x-axis and the set of points on the y-axis.

Page 104.

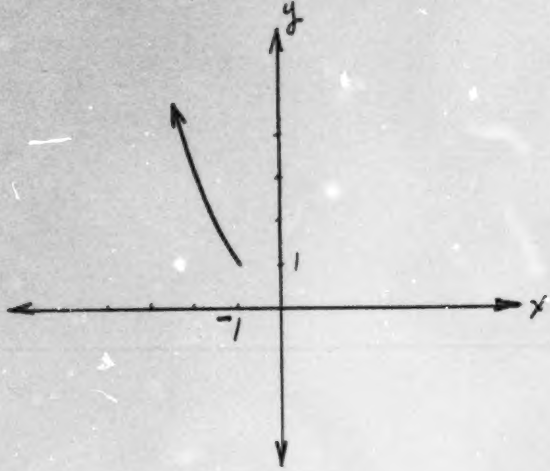
2. (a)



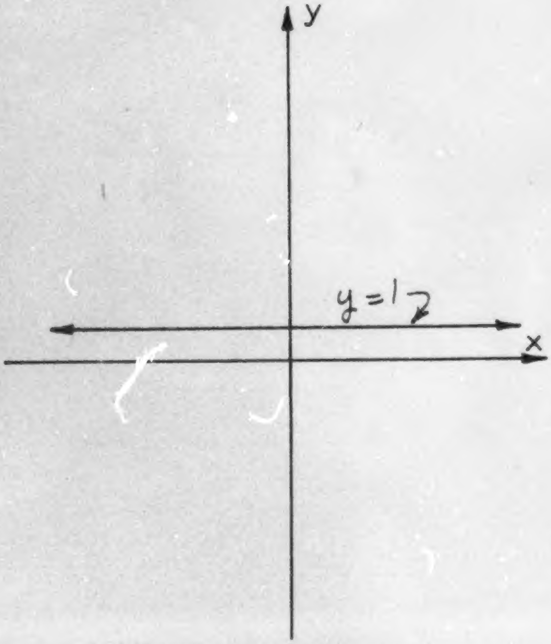
(b)



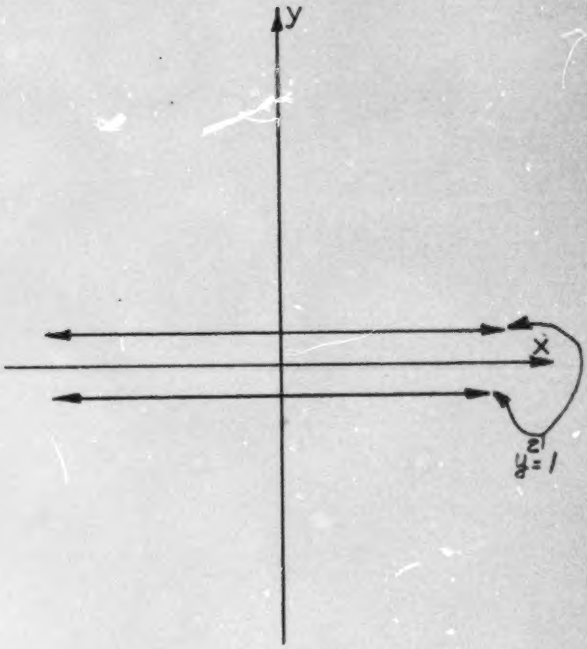
(c)



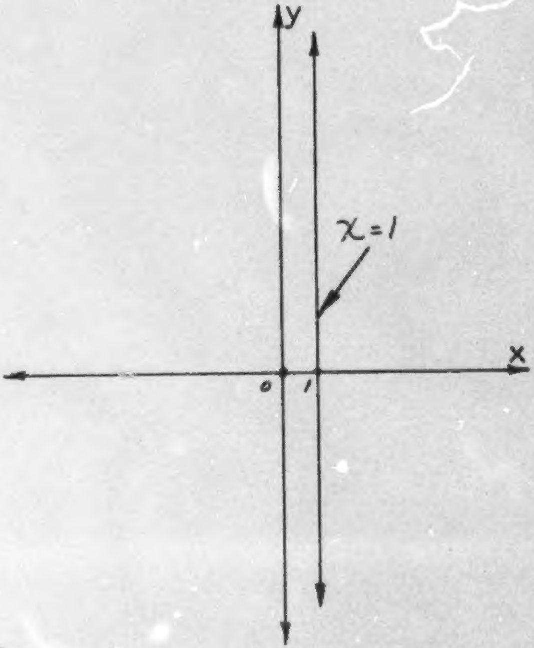
(d)



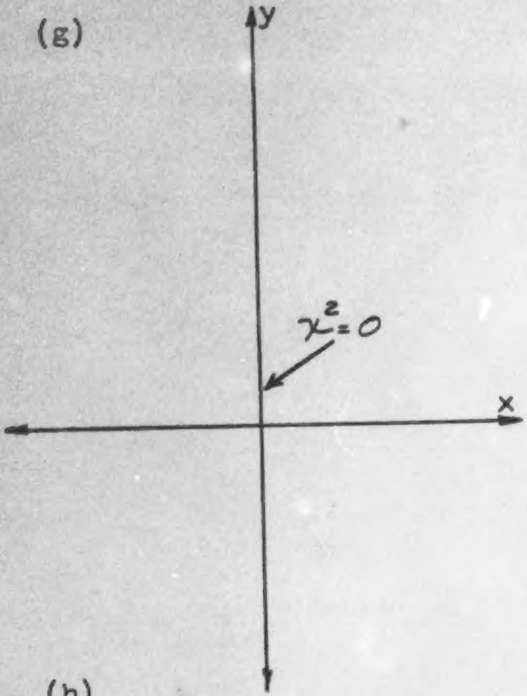
(e)



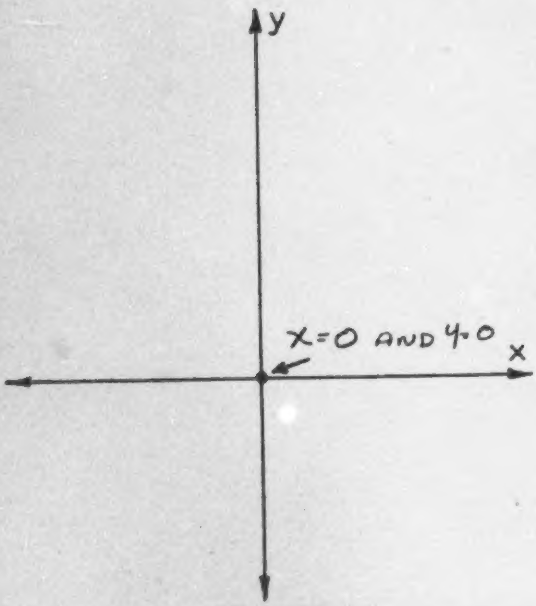
(f)



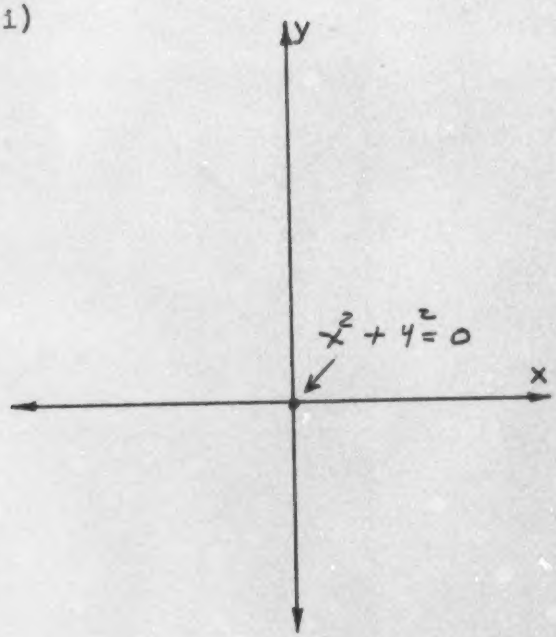
(g)



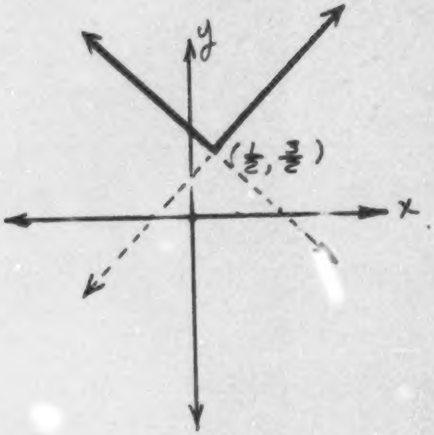
(h)



(i)

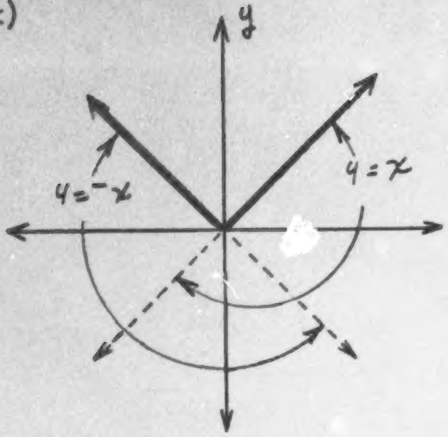


*(j)


y
x
$(\frac{1}{2}, \frac{3}{2})$

Graphs of $y = x + 1$ and $y = 2 - 2$ shown. Graph of the number sentence in 3(j) is the union of the two darkened rays.

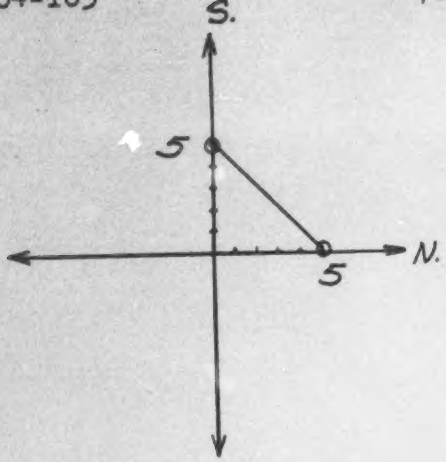
*(k)



The graph of the number sentence is the union of the two darkened rays.

3.	(a)	(0,1), (1,0)	2 solutions
	(b)	(0,2), (1,1), (2,0)	3 solutions
	(c)	(0,20), (1,19), (2,18), (3,17), (4,16), (5,15), (6,14), (7,13), (8,12), (9,11), (10,10), (11,9), (12,8), (13,7), (14,6), (15,5) (16,4), (17,3), (18,2), (19,1) (20,0)	21 solutions
	(d)	(0,0)	1 solution
	(e)	(1,0)	1 solution
	(f)	(2,0), (0,1)	2 solutions
	(g)	(3,0), (1,1)	2 solutions
	(h)	(4,0), (2,1), (0,2)	3 solutions
	(i)	(1,12), (3,11), (5,10), (7,9), (9,8), (11,7),(13,6), (15,5), (17,4), (19,3), (21,2), (23,1) (25,0)	13 solutions
	(j)	(0,5), (7,0)	2 solutions
	(k)	(3,3)	1 solution
	(l)	(6,1)	1 solution

4.



5.

Bank A: $i = 4t$

Bank B: $i = 5t$

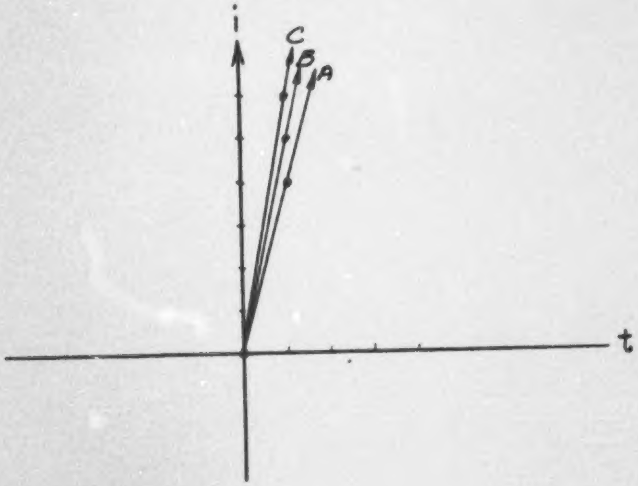
Bank C: $i = 6t$

The number sentences are:

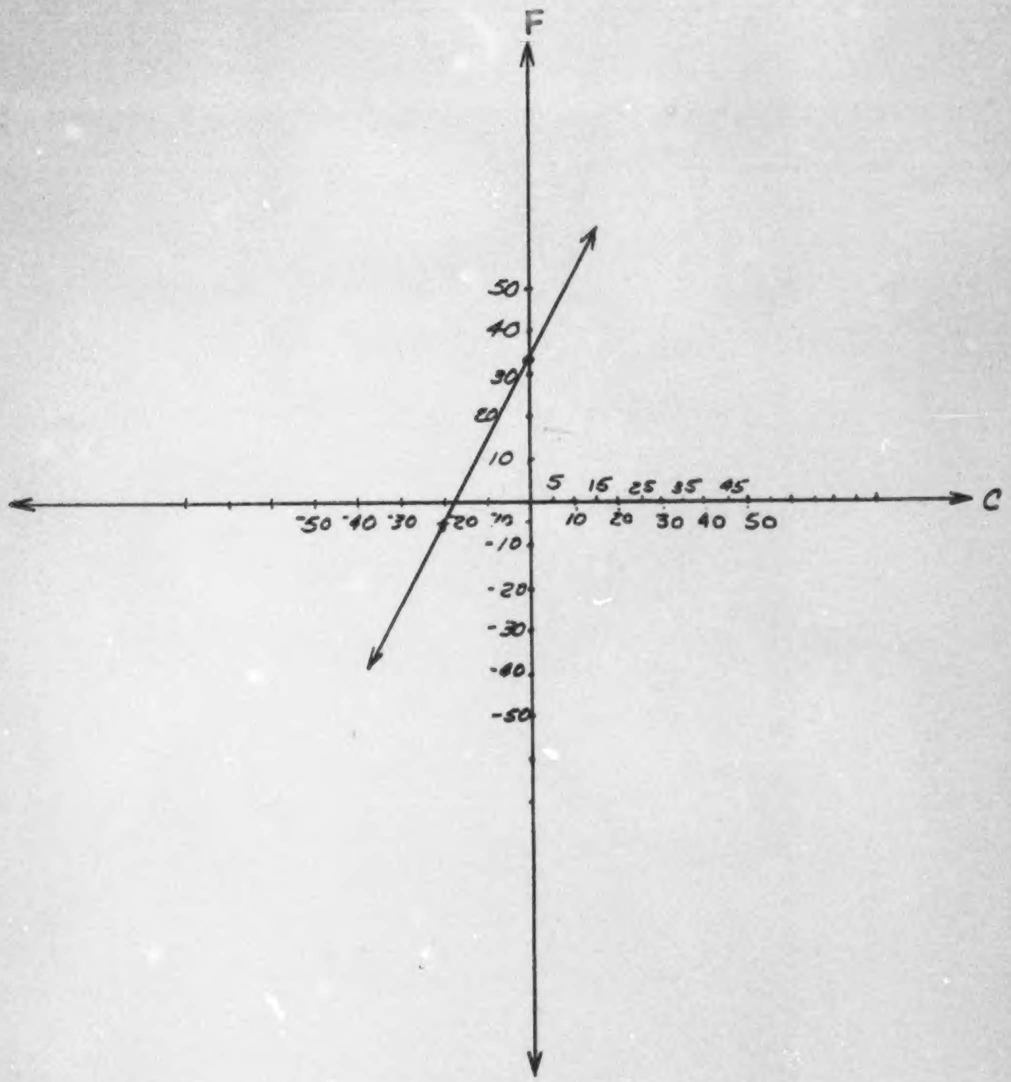
"$i = 4t$ and $i > 0,\ t > 0$"

"$i = 5t$ and $i > 0,\ t > 0$"

"$i = 6t$ and $i > 0,\ t > 0$"



6.



(a) If C = $^{-}25$ then F = $^{-}13$.
If C = $^{-}15$ then F = 7.
If C = 0 then F = 32
If C = 4 then F = $\frac{196}{5} = 39\frac{1}{5}$.

(b) If F = $^{-}30$ then C = $^{-}(\frac{310}{9}) = {}^{-}34\frac{4}{9}$
If F = $^{-}15$ then C = $^{-}(\frac{235}{9}) = {}^{-}26\frac{1}{9}$
If F = 0 then C = $^{-}(\frac{160}{9}) = {}^{-}17\frac{7}{9}$

Page 106.

7. The temperature is $^-40$ degrees. Edward had said that $F = C$, and we know that $F = \frac{9}{5}C + 32$. Thus $C = \frac{9}{5}C + 32$. Adding $^-(\frac{9}{5})C$, using the addition property, we have

$$C + {}^-(\tfrac{9}{5})C = {}^-(\tfrac{9}{5})C + (\tfrac{9}{5}C + 32)$$

or $$(1 + {}^-(\tfrac{9}{5}))C = 32$$

or $${}^-(\tfrac{4}{5})C = 32$$

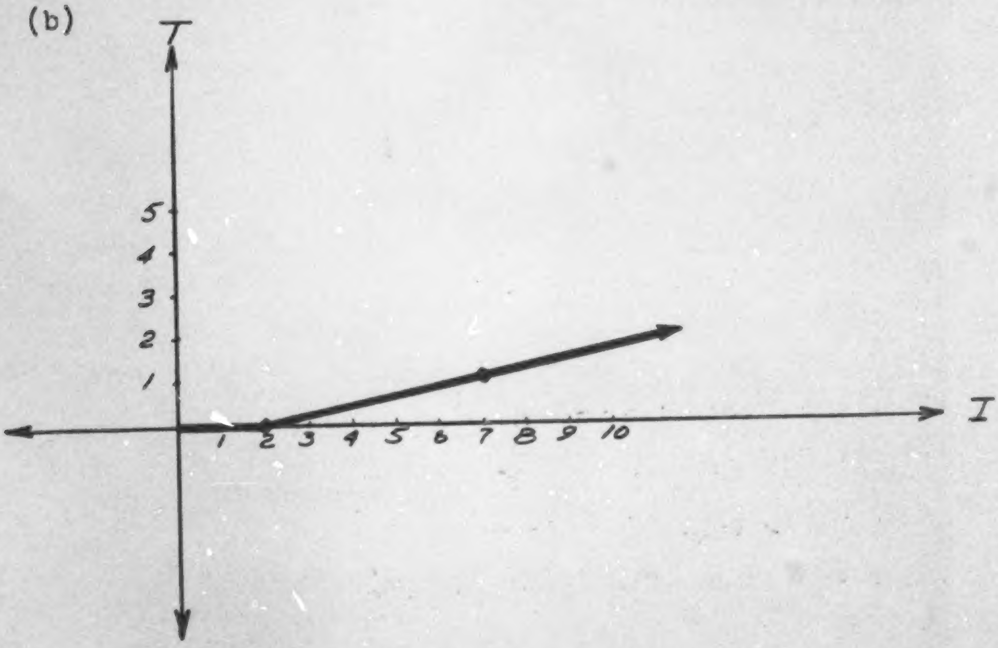
Multiplying by $^-(\frac{5}{4})$ using the multiplication property, we have

$$C = {}^-(\tfrac{5}{4}) \cdot 32 = \frac{^-5 \cdot 32}{4} = \frac{^-5 \cdot 8 \cdot 4}{1 \cdot 4} = \frac{^-5 \cdot 8}{1} = {}^-40$$

Checking to see whether $^-40$ is a solution: If $C = {}^-40$ then $\frac{9}{5}C + 32 = \frac{9}{5}({}^-40) + 32 = 9({}^-8) + 32 = {}^-72 + 32 = {}^-40$. Thus $C = {}^-40 = \frac{9}{5}C + 32$, so $^-40$ is a solution.

8. (a) "T = the larger of the numbers $(.2)I - .4$ and 0, and $I \geq 0$"

(b)



(c) If $I = 10$ then $T = 1.6$, so the tax on an income of \$10,000 is \$1600.
If $I = 3.5$ then $T = .3$, so the tax on an income of \$3500 is \$300.
If $I = 1.5$ then $T = 0$, so the tax on an income of \$1500 is \$0.
If $T = 1.5$, then $I = 9.5$, so that his income is \$9500.

Sample Questions for Chapter 2.

PartI: True - False

True 1. If two numbers are unequal, then one must be greater than the other.

False 2. "Open sentence" is another name for a "mathematical phrase."

False 3. The solution set of an open sentence must contain at least one element. (Note: the idea of an empty or vacuous set is not discussed in the text for pupils but is touched upon in the Teachers' Commentary.)

True 4. $4(5 - 2)$ and $\frac{(6 \cdot 2)}{1}$ are numerals for the same number.

False 5. $3(\frac{2}{3} + 5) < 17$.

False 6. $\frac{4}{2} \cdot 0 = 8(\frac{1}{4})$.

True 7. A phrase is a part of a sentence.

True 8. The expression $x + 5$ is called an open phrase.

True 9. The expression "$(6a + 2b + 3c)$" represents or names one number.

False 10. The addition property tells us that equals may be added to equals.

True 11. "$4 + 2 = 7$" is an equation.

False 12. "=", "<", and "+" are verbs in number sentences.

False 13. If the solution set of $x \div 1 = 1 \div x$ is represented on the number line, the representation is the entire number line.

True 14. Parentheses tell us to do the operation within them first.

False 15. $\frac{x}{x} = 1$ is true for all values of x. (Note to teacher: $\frac{0}{0}$ is not possible).

True 16. $a(3 + 4) = (4 + 3)a$ for all values of a.

False 17. Any mathematical expression consisting of two number phrases is a number sentence.

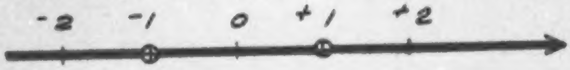
True 18. $(7 + 4)z = z(5 + 6)$ for all values of z.

True 19. A mathematical inequality may also be called a number sentence.

False 20. If $x = {}^{-}10$, then $2x + 8 = 12$.

False 21. Any number sentence may be called an equation.

False 22. The representation shown on the following number line



indicates the set of solutions $\{{}^{-}1, {}^{+}1\}$.

For True-False Items 23-27: Determine if the open sentences are true for the indicated values for the unknown number.

False 23. $x + 0 = 4x$, if $x = 3$

True 24. $\frac{y + 8}{3} = \frac{\frac{16}{y}}{1}$, if $y = 4$

True 25. $5(z + 1) > 5z$, if $z = 3$

True 26. $\frac{z}{5} < \frac{z}{4}$, if $z = 8$

False 27. $(x + y)(x - y) = (x + y)(x + y)$, if $x = 5$ and $y = 2$

Part II: Multiple-Choice

For Multiple-Choice items 1-3 select the answer which correctly represents the series of operations.

C 1. Multiply the difference of eight and two by three, then add four to the product and divide this sum by two.

A. $8 - 2 \cdot 3 + 4 \div 2$

B. $(8 - 2)3 + 4 \div 2$

C. $\frac{3(8 - 2) + 4}{2}$

D. $\frac{3(8) - 2 + 4}{2}$

E. $3(8 - 2) + (4 \div 2)$

B 2. Double the sum of three and eight and then subtract seven from the product.

A. $2(3) + 8 - 7$

B. $2(3 + 8) - 7$

C. $2(3 + 8 - 7)$

D. $2(3) + (8 - 7)$

E. $2 \cdot 3 + 8 - 7$

A 3. The set of solutions for the open sentence $8 + c < 12$ is

A. All numbers less than 4.

B. All numbers greater than 4.

C. 4.

D. All numbers except 4.

E. None of the above is correct.

D 4. Which one of the following number sentences is an inequality?

A. $17 > 2 + 2$

B. $3 < 2 + 2$

C. $x \div .10 > 0.1$

D. All of the above answers are correct.

E. None of the above answers are correct.

C 5. The phrase $2 \cdot (5) + 4$ represents which one of the following?

A. 8 B. 10 C. 14 D. 18 E. 40

In questions 6-19 a statement of a mathematical situation has been given in words. Following each are five mathematical phrases or sentences. You are to select the phrase or sentence which best represents the stated situation.

A. 6. My age seven years from now. (x = number of years in my present age.)

A. $7 + x$

B. $7 - x$

C. $7(x)$

D. $x - 7$

E. $\frac{7x}{2}$

C 7. The perimeter of a rectangle whose width is one-half its length. (y = number of inches in the width).

A. $(2y)y$
B. $2y + y$
C. $(2y + y)2$
D. $\frac{1}{2}y + y$
E. $3y$

B 8. The area of a square whose side is s.

A. $2s$
B. $s \cdot s$
C. $4s$
D. $s \cdot 2s$
E. $s + 4$

C 9. A number, plus four times the number, is sixty. (x = the number).

A. $4x = 60$
B. $x + 4 = 60$
C. $x + 4x = 60$
D. $60 - 4x = 4$
E. $x = 60(4)$

D 10. If I spend one-half the amount of money I have, I will have less than seven dollars. (a = the amount of money I have).

A. $2a > 7$
B. $\frac{1}{2}a = 7$
C. $a - 2 < 7$
D. $a - \frac{a}{2} < 7$
E. $a > 14$

E 11. I am twice as old as you are. In three years the sum of our ages will be 27. (x = the number of years in your age).

A. $2x + x = 27$
B. $2x + x = 24$
C. $(2x - 3) + (x + 3) = 27$
D. $(x - 3) + (2x - 3) = 27$
E. $(x + 3) + (2x + 3) = 27$

A 12 In my pocket I have $3.90. I have a certain number of nickels. I have three times as many dimes as I do nickels and six more quarters than nickels. (n equals the number of nickels).

A. $5n + 10(3n) + 25(n + 6) = 390$
B. $n + 3n + n + 6 = 390$
C. $5n + 6 = 390$
D. $5n + 10n + 6 = 390$
E. $5n + 10(4 + n) + 25(6n) = 390$

C 13. Ten yards of cloth will cost more than 12 dollars. (x = the cost per yard in dollars).

A. $x > 12$
B. $x + 10 > 12$
C. $10x > 12$
D. $\frac{10}{x} > 12$
E. $\frac{x}{10} > 12$

E 14. The length of a rectangle is twice its width and its area is 18 square inches. (x = the number of inches in the length).

A. $2x(x) = 18$
B. $2x + x = 18$
C. $x(x + 2) = 18$
D. $2x = 18$
E. $x(\frac{x}{2}) = 18$

B 15. John is three years less than twice Bill's age. Together their ages are greater than 25 years. (x = number of years in Bill's age).

A. $(2x + 3) - x > 25$
B. $x + (2x - 3) > 25$
C. $x + (2x + 3) > 25$
D. $x(2x + 3) > 25$
E. $x - (2x + 3) > 25$

C 16. A boat goes downstream 10 miles per hour faster than the rate of the current. Its speed downstream is 25 miles per hour. How fast is the current? (r = rate of current).

A. $\frac{r}{10} = 30$
B. $\frac{10}{r} = 25$
C. $r + 10 = 25$
D. $r - 10 = 30$
E. $r(10) = 25$

A 17. A two-digit number is 7 more than 3 times the sum of the digits. (t is the ten's digit and u is the unit's digit).

A. $10t + u = 3(u + t) + 7$
B. $tu - 7 = 3(t + u)$
C. $tu + 7 = 3tu$
D. $10t(u) = 3(t + u)$
E. $10t + u = 3(t + u) - 7$

B 18. A man is 9 times as old as his son. In 3 years the father will be only 5 times as old. What is the age of each? (x = number of years in the son's age).

A. $9x + 3 = 5x$

B. $9x + 3 = 5(x + 3)$

C. $9x - 3 = 5(x - 3)$

D. $12x = 8x$

E. $9x = 5x + 3$

A 19. The sum of two numbers is 40 and the larger exceeds three times the smaller by 4. (The smaller number is x).

A. $x + 3x + 4 = 40$

B. $x + 3x - 4 = 40$

C. $x - 4 + 3x = 40$

D. $x + 3x = 44$

E. $x - 3x + 4 = 40$

In questions 20-23 an open sentence is written. Following are four English sentences. Select the answer which indicates the English sentence or sentences which correspond to the open sentence.

C 20. $y > 10$

I. I am more than ten years old.

II. John has ten dollars.

III. The board is over ten inches long.

IV. There are a number of marbles in the bag.

A. Only I and II are correct.

B. Only II and III are correct.

C. Only I and III are correct.

D. Only III and IV are correct.

E. Only I, III, and IV are correct.

E 21. $x < 360$

I. A year contains more than 360 days.

II. 360 degrees is one rotation.

III. Bob has \$3.60.

IV. The area of the square is 360 square inches.

A. Only I is correct.

B. Only II is correct.

C. Only III is correct.

D. Only IV is correct.

E. None of the answers is correct.

A 22. $K < 13$

I. The temperature is below thirteen degrees.

II. Joan bought more than thirteen pairs of shoes.

III. There are fewer than 13 books overdue.

IV. Bill has thirteen dollars.

A. Only I and III are correct.
B. Only II and IV are correct.
C. Only I, II, and III are correct.
D. Only II, III and IV are correct.
E. All of the answers are correct.

D 23. $3a = 4b$

I. My salary quadrupled is matched by Jack's salary tripled.

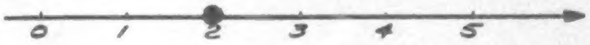
II. There are four more bananas when the apples are increased by three.

III. The perimeters of an equilateral triangle and a square are equal.

IV. Three times a certain whole number is equal to four times a number.

A. All of the answers are correct.
B. Only I, II, and III are correct.
C. Only II, III and IV are correct.
D. Only I, III and IV are correct.
E. None of the answers is correct.

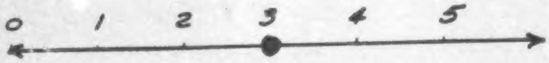
D 24. Which one of the number sentences below has its solution set represented on the number line in the drawing?

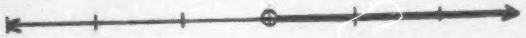


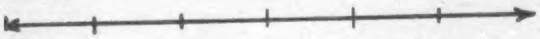
I. $x = 2$
II. $2x = x + 2$
III. $\frac{x}{2} = 1$
IV. $x - 2 = 0$

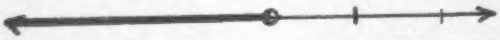
A. Only I is correct.
B. Only II is correct.
C. Only III is correct.
D. All of the sentences are correct.
E. None of the sentences are correct.

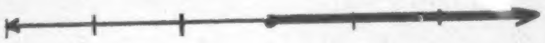
B 25. Which one of the representations on the number line below represents the set of solutions of the number sentence $x > 3$?

A. 

B. 

C. 

D. 

E. 

B 26. Which of the following sets is the solution set of the equation "$x + 7 = 3$"?

A. $\{4\}$
B. $\{^-4\}$
C. $\{4, {}^-4\}$
D. $\{10\}$
E. None of the above.

E 27. Which of the following sets is the solution set of the sentence "$1 - x > 2$"?

A. The set of all numbers greater than 3.
B. The set of all numbers less than 1.
C. The set of all numbers greater than $^-1$.
D. The set of all numbers greater than $^-3$.
E. The set of all numbers less than $^-1$.

D 28. Which of the following sets is the solution set of the sentence "$x - 1 < 0$ and $2x - 1 = 3$"?

A. The set of all numbers less than 1.
B. $\{2\}$
C. The set of all numbers between 1 and 2.
D. The empty set.
E. $\{1,2\}$

C 29. Which of the following sets is the set of solutions for the sentence "$x + 3 > 0$ and $5 - x > 1$"?

A. The set of all numbers between 1 and 3.

B. The set of all numbers greater than $^{-}3$.

C. The set of all numbers between $^{-}3$ and 4.

D. The empty set.

E. The set of all numbers less than 4.

Part III-A Matching.

Select the items from the column on the right which match the items in the column on the left. Select an item which best describes the computation shown for each problem. NOTE: the items in the right column may be used once, several times if needed or not at all. Fill in the space indicated by the letter of your choice on the answer sheet.

C	1.	A number diminished by 3.	A.	$2x + 1$
B	2.	The temperature rising three degrees.	B.	$x + 3$
D	3.	The cost of x pencils at 3 cents each.	C.	$x - 3$
D	4.	A number increased by twice the number	D.	$3x$
E	5.	The number of yards in x feet.	E.	$\frac{x}{3}$

Part III-B. Matching. (6-10)

(Follow directions given for Part A.)

C	6.	The amount of money in my pocket: t dimes and u pennies	A.	$t + u$
A	7.	A number increased by another number.	B.	$t - u$
E	8.	The ratio of two numbers.	C.	$10t + u$
A	9.	The sum of the digits of a two-digit number.	D.	tu
C	10.	A two-digit number.	E.	$\frac{t}{u}$

Part III-C. Matching. (11-15).

(Follow directions given for Part A.)

E	11.	John had twenty marbles. He gave some marbles to Bob and had nine left. How many marbles did he give Bob?	A.	$x + 9 = 20$
A	12.	Mary will be twenty years old nine years from now.	B.	$x - 20 < 9$
C	13.	After losing nine dollars, Allan deposited the rest of his money. The deposit was more than $20. How much money did Allan have?	C.	$x - 9 > 20$
C	14.	From what numbers can you subtract nine so that each difference is greater than twenty?	D.	$x + 9 < 20$
A	15.	In a class of 20 pupils, nine pupils received scores of A on the test. How many pupils had scores other than A?	E.	$20 - x = 9$

Chapter 3

SCIENTIFIC NOTATION, DECIMALS, AND THE METRIC SYSTEM

For this chapter it is assumed that the student has had some acquaintance with the names of numbers, the decimal notation, and finding products involving decimals and per cents.

It is intended that the class exercises be done during class time. The procedure for getting each answer should be discussed before continuing to the next problem.

This chapter should take about 13 days.

3-1. Large Numbers

Page 107. This section seeks to cultivate the ability to read large numbers, an appreciation of them, and ability to write them in scientific notation. Some people prefer to use the term "standard form" instead of "scientific notation." A certain amount of estimation is also included.

Page 108. Of course there is no largest number and two knowledgeable boys would play the game described to a draw by the simple process of adding one, multiplying by two or in some other way increasing the number which their opponent had just given. The British system requires fewer names since it groups the numbers by millions but one must hold his breath longer in saying a number. (Perhaps the British have better lungs.)

Page 110. Some of the more thoughtful students may wonder why we do not write 93 million, for instance as 93×10^6 where the exponent is used to indicate the number of zeros in the numeral. There is no point in trying to hide the fact that in many cases this is really a little simpler, and there is no reason to try to prevent students from using it (see comments on Section 2). But two things should be made clear: in the first place, this is the notation which the scientists use; and, second, in the use of logarithms, the scientific notation is certainly much simpler.

Answers to Class Exercises 3-1-- Pages 109-111

1. (a) 10^9
   (b) 10^{12}
   (c) 10^{15}

2. (a) 7×10^3 (d) 14×10^6
   (b) 5×10^4 (e) 375×10^6
   (c) 3×10^6 (f) 48×10^{10}

   It is also correct to have other answers than the above. For example, (b) might be expressed as 50×10^3; 500×10^2, 5000×10^1, or even $50,000 \times 10^0$. Some of these answers will undoubtedly appear during the class discussion, and their appearance should be encouraged.

3. (a) 7.6×10 (f) 4.83259×10^3
   (b) 8.59×10^2 (g) 8.412×10^2
   (c) 7.623×10^3 (h) 9.7836×10^3
   (d) 8.463×10^6 (i) 3.412789435×10^6
   (e) 7.648×10 (j) 5.36425×10^{10}

4. (a) No; because 15 is not between 1 and 10.
   (b) Yes. Fits the definition.
   (c) No; because 0.12 is not between 1 and 10.

5. (a) 5.687×10^3
   (b) 1.4×10
   (c) 3.5×10^6

6. (a) 3,700,000 (b) 470,000 (c) 5,721,000

7. (a) 9.3×10^7
   (b) 1,395,000 or 1.395×10^6
   (c) 91,605,000
   (d) 94,395,000
   (e) 9.1605×10^7 and 9.4395×10^7

Answers to Exercises 3-1. Pages 111,112.

1. (a) 10^3
   (b) 10^{10}
   (c) 10^4
   (d) 10^9
   (e) 10^6
   (f) 10^7

2. (a) 6×10^3
   (b) 6.78×10^2
   (c) 9×10^9
   (d) 4.59×10^8
   (e) 7.8×10^4
   (f) 6×10^3
   (g) 1.56×10^4
   (h) 7.81×10^9

3. (a) 100,000
   (b) 583
   (c) 30,000
   (d) 50,000,000
   (e) 630
   (f) 820,010,000
   (g) 436,000,000
   (h) 1,732,400

4. (a) Seven hundred eighty-three
   (b) Seven million, five hundred thousand
   (c) Six hundred thirty-two thousand, seven
   (d) Three hundred sixty-two and three hundred sixty-two thousandths.
   (e) Four billion, two hundred eighty-four thousand, six hundred thirty-two.
   (f) Four and two thousand five hundred six thousandths.

5. 10^3 or 3^{10}

6. (a) $600 = 6 \times 10^2$
   (b) $100 = 10^2$
   (c) $1200 = 1.2 \times 10^3$
   (d) $70900 = 7.09 \times 10^4$
   (e) $600,000 = 6 \times 10^5$
   (f) $5,362,400 = 5.3624 \times 10^6$

7. 10^2

3-2. Calculating with Large Numbers.

Pages 112,113. The objectives of this section are a continuation of those in the first with the added skill of multiplying, using the scientific notation.

Here again, some students may prefer to multiply 93,000,000 by 11,000, for example, by writing the former as

93×10^6 and the latter as 11×10^3, then forming the product: 1023×10^9 and then putting it into scientific notation: 1.023×10^{12}. The teacher may prefer this, too, in which case he should use it.

To see that $1.023 \times 10^3 = 1023$ one could refer to the rules for multiplying decimals or the teacher might prefer to go back to first principles. One way to do the latter would be:

$$1.023 = 1 + \frac{23}{1000} \text{ and, since } 10^3 = 1000,$$

$$1.023 \times 10^3 = (1 + \frac{23}{1000}) \times 1000$$

$$= 1000 + \frac{23}{1000} \times 1000 = 1000 + 23 = 1023.$$

However, the student should realize that multiplying a number by 10 is equivalent to moving each digit one place to the left, or simply moving the decimal point one place to the right. He should, however, be able to show on demand why either is so.

In Problem 3 (Page 115)notice that the speed of the space ship is about a mile per second, which is about 32 million miles a year.

Answers to Class Exercises 3-2. Pages 114,115

1. (a) 60 × 186,000
   (b) 60 × 60 × 186,000
   (c) 24 × 60 × 60 × 186,000
   (d) 365 × 24 × 60 × 60 × 186,000
   (e) Do not "round off" until multiplications have been performed; if "rounding off" takes place earlier the result becomes less accurate. Before rounding it is 6,307,200,000,000.
   (f) trillion
   (g) The speed of light used is the result of rounding an approximation. The number of days in a year (365) is an approximation also.

Answers to Exercises 3-2. Pages 114,115.

1. (a) 6×10^{10} (e) 7.63×10^{7}
(b) 1.2×10^{19} (f) 2.16×10^{5}
(c) 3.5×10^{13} (g) 2.1×10^{6}
(d) 6×10^{8} (h) 9.3×10^{14}

2. (a) 6.3×10^{11} (c) 4.65×10^{9}
(b) 10^{9} (d) 1.1×10^{9}

3. (a) 1.728×10^{4} miles (c) 4.32×10^{6} miles
(b) 7.2×10^{4} miles (d) 6.3072×10^{7}miles
No, this distance is about 93 million miles
(or 9.3×10^{7}miles).

4. (a) 4×10^{7}
(b) 2×10^{10}
(c) 1.2×10^{6}

5. 7.2×10^{9} miles

6. Yes, you could have made 63,072,000 marks.

7. About 6.132×10^{8} miles or 613,200,000 miles.

3-3. Small Numbers.

Answers to Class Exercises 3-3. Page 117.

1. (a) 10^{-4} (d) 10^{-9}
(b) 10^{-6} (e) 10^{-5}
(c) 10^{-7}

2. (a) $\frac{1}{10^{3}}$ or $\frac{1}{1000}$ (c) $\frac{1}{10^{7}}$ or $\frac{1}{10,000,000}$
(b) $\frac{1}{10^{5}}$ or $\frac{1}{100,000}$ (d) $\frac{1}{10^{6}}$ or $\frac{1}{1,000,000}$

Answers to Exercises 3-3. Page 118.

1. (a) 9.3×10^{-2}

(b) 10^{-4}

(c) 10^{-6}

(d) 10^{0}
(e) 6.21×10^{-3}
(f) 10^{-2}
(g) 7.006×10^{-1}
(h) 9.07×10^{-7}
(i) 6×10^{0}
(j) 4.5×10^{-3}

Page 119.

2. (a) 0.000093
(b) 0.107
(c) 0.000001
(d) 0.0005
(e) 0.007065
(f) 0.1
(g) 0.00000143
(h) 0.00038576

3. (a) 6.3×10^{5}
(b) 1.57×10^{-4}
(c) 2.4×10^{-6}
(d) 5.265×10^{-5}
(e) 3.6235×10^{2}
(f) 4.32×10^{-3}
(g) 3.05×10^{-9}
(h) 6.95×10^{0}

4. (a) $^{-}3$
(b) 6.3
(c) $^{-}7$
(d) 5
(e) $^{-}1$
(f) $^{-}3$
(g) 21300
(h) 0.0000213

5. The number zero, since a product is zero only when one of the factors is zero. Neither a "power of ten" nor "a number between 1 and 10" can be zero.

3-4. Multiplication: Large and Small Numbers.

Answers to Exercises 3-4. Page 121.

1. (a) 10^{-7}
(b) 3×10^{-3}
(c) 10^{-13}
(d) 8×10^{-5}
(e) 7×10^{-7}
(f) 5.7×10^{-10}
(g) 10^{24}
(h) 10^{-3}

2. (a) 2.38×10^{-8} (d) 3×10^{-10}
(b) 5.4×10^{-9} (e) 4.56×10^{-6}
(c) 1.4×10^{-7} (f) 5.6896×10^{-6}

3. (a) 10^{2} (c) 10^{-6}
(b) 10^{2} (d) 10^{-7}

4. 3.4594×10^{-1}

5. 4.125×10^{5} dollars

6. 9.939×10^{9} dollars

7. 1.08×10^{6} grams

3-5. Division: Large and Small Numbers.

Answers to Class Exercises 3-5. Page 123.

1. (a) 10^{-9} (b) 10^{5}

2. (a) 10^{5} (b) Because 7-2 equals 5
(c) The same number

3. (a) 9 (b) 10^{9} (c) Both equal 10^{9}

4. (a) 10^{-9} (b) Yes. Both equal 10^{-9}.

5. Yes. Each side equals 10^{5}.

6. $10^{a - b}$

7. (a) 10^{16} (b) 10^{1} (c) 10^{-12}

8. Yes. Yes.

9. (a) 3×10^{-11} (b) 2×10^{-1} (c) 3×10^{7}

Answers to Exercises 3-5. Page 124.

1. (a) 10^{3} (d) 10^{5} (g) 10^{-6}
(b) 10^{2} (e) 10^{-2} (h) 10^{-1}
(c) 10^{10} (f) 10^{-10}

3-6. Use of Exponents in Multiplying and Dividing Decimals.

Page 126. Note that here division is handled somewhat differently from the discussion in Chapter 8 of Volume I. Powers of 10 are used in such a fashion that in the formal division portion a whole number is divided by a whole number. Many people find this easier to follow than the procedure which uses a decimal as the dividend.

There are too many problems in the exercises to be given as one assignment. Some of the students may need the extra practice and will need two or more days.

Answers to Class Exercises 3-6. Page 127.

1. (a)

$$\begin{array}{rcrl} 6.14 & = & 614 & \times 10^{-2} \\ \times\ \underline{0.42} & =\times & \underline{42} & \times \underline{10^{-2}} \\ & & 1228 & \\ & & \underline{2456} & \\ & & 25788 & \times 10^{-4} = 2.5788 \end{array}$$

(b)

$$\begin{array}{rcrl} 0.625 & = & 625 & \times 10^{-3} \\ \times\ \underline{0.038} & =\times & \underline{38} & \times \underline{10^{-3}} \\ & & 5000 & \\ & & \underline{1875} & \\ & & 23750 & \times 10^{-6} = 0.023750 \end{array}$$

(c)

$$\begin{array}{rcrl} 649.3 & = & 6493 & \times 10^{-1} \\ \times\ \underline{14.68} & = & \underline{1468} & \times \underline{10^{-2}} \\ & & 51944 & \\ & & 38958 & \\ & & 25972 & \\ & & \underline{6493} & \\ & & 9531724 & \times 10^{-3} = 9531.724 \end{array}$$

(d)

$$\begin{array}{rcrl} 11.4 & = & 114 & \times 10^{-1} \\ \times\ \underline{0.0031} & & \underline{31} & \times \underline{10^{-4}} \\ & & 114 & \\ & & \underline{342} & \\ & & 3534 & \times 10^{-5} = 0.03534 \end{array}$$

2. (a) 10^7
(b) 10^4
(c) 10^{18}
(d) 10^{29}
(e) 10^{24}
(f) 10^{30}
(g) 10^{18}
(h) 10^7

3. (a) 10^{-7}
(b) 10^{-4}
(c) 10^{-18}
(d) 10^{-29}
(e) 10^{-24}
(f) 10^{-30}
(g) 10^{-18}
(h) 10^{-7}

4. (a) 10^{-3}
(b) 10^{-10}
(c) 10^2
(d) 10^6
(e) 10^{-2}
(f) 10^{-5}
(g) 10^{10}
(h) 10^1

5. (a) 2×10^{-3}
(b) 7×10^{-7}
(c) 1.2×10^9
(d) 2.4×10^2
(e) 4×10^{-2}
(f) 4×10^{-3}

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6. (a) $^-2$ and $^-1$
(b) $^-2$ and $^-1$
(c) 2, $^-2$ and $^-3$
(d) $^-2$ and 0
(e) $^-2$, 2 and 4
(f) $^-2$, 3, $^-2$ and 5

7. 9.2×10^7. Treat as problem in equations,

$$\frac{3}{100}y = 2.76 \times 10^6$$

8. No, since it will take 100 years to spend this sum of money.

9. About 1180 days. (Rounded to nearest ten)

10. $20,000. Treat as problem in ratio.

11. 40%. Treat as problem in equations,

$$14 \times 10^6 = \frac{y}{100}(35 \times 10^6)$$

Answers to Exercises 3-6. Pages 128-129.

1. (a) 0.18063 (c) 7500 (e) 7.2
   (b) 0.0684 (d) 399.529
2. (a) $^{-}2$ (c) 4 (e) 63700
   (b) $^{-}3$ (d) $^{-}3$ (f) 2,1
   (g) 0.0412
3. (a) 300200 (c) 175.02 (e) 160
   (b) 6.1 (d) 0.000007
4. (a) $135 \times 6 \times 10^{-2} = 810 \times 10^{-2} = 8.1$
   (b) $(76 \times 10^{3}) \times (3 \times 10^{3}) = 228 \times 10^{6} = 228{,}000{,}000$
   (c) $(18 \times 10^{3}) \times (3 \times 10^{-4}) = 54 \times 10^{-1} = 5.4$
   (d) $(35 \times 10^{-4}) \times (16{,}301 \times 10^{-3}) = 570535 \times 10^{-7} = 0.0570535$
   (e) $(6 \times 10^{6}) \times (275 \times 10^{-4}) = 1650 \times 10^{2} = 165{,}000$
   (f) $(7 \times 10^{-2}) \times (3 \times 10^{2}) \times (2 \times 10^{-2}) \times (6 \times 10^{3}) = 252 \times 10^{1} = 2520$
5. (a) $(63 \times 10^{-1}) \div (3 \times 10^{-1}) = 21$
   (b) $(78 \times 10^{-2}) \div (13 \times 10^{0}) = 6 \times 10^{-2} = 0.06$
   (c) $\dfrac{8750 \times 10^{0}}{875 \times 10^{-2}} = 10 \times 10^{2} = 1000$
   (d) $\dfrac{1470 \times 10^{-4}}{75 \times 10^{-2}} = 19.6 \times 10^{-2} = 0.196$
   (e) $27 \times 10^{-2}\,\overline{\big)\,84402 \times 10^{-5}} = 3126 \times 10^{-3} = 3.126$
   (f) $18 \times 10^{2}\,\overline{\big)\,216 \times 10^{-1}} = 12 \times 10^{-3} = 0.012$
6. $\dfrac{4186 \times 10^{-1} \times 19 \times 10^{-3}}{13 \times 10^{-2}} = 6118 \times 10^{-2} = 61.18$

7. $\frac{6 \times 840}{.04} = \frac{5040 \times 10^0}{4 \times 10^2} = 1260 \times 10^2 = 126000$

It is necessary to make 126,000 pieces.

8. $\frac{5\frac{1}{3} \times (6.3 \times 10^{12})}{100{,}000 \times (3.2 \times 10^7)} = \frac{\frac{16}{3} \times (63 \times 10^{13})}{10^5 \times 32 \times 10^8} = \frac{16 \times 21}{32} = 10.5$

It will take about 10.5 years to go one way, 21 years round trip.

3-7. The Metric System.

Pages 129, 131. "The invention of the Hindu-Arabic decimal number system is one of man's outstanding achievements. With it, for the first time in history, masses were able to learn the art of computation. Later Simon Stevin still further simplified the processes of computation by the introduction of the decimal fraction. Today, the decimal fraction should be called the common fraction, so widely is it used in commerce and technology.

"Still later came the metric system of measures, based upon the units, meter, liter, and gram, which are also decimal. ...If the selection of a system of measures were optional with educators, they would unhesitatingly choose a decimal system. They are well aware of the tremendous efforts required to learn, for example, the relationship between linear units in our system: 1 inch $= \frac{1}{12}$ foot, 1 foot $= \frac{1}{3}$ yard, 1 yard $= \frac{2}{11}$ rod, 1 rod $= \frac{1}{320}$ mile. In contrast, they appreciate the simplicity and ease with which the pupil could learn: 1 millimeter = 0.1 centimeter, 1 centimeter = 0.01 meter and 1 meter = 0.001 kilometer.

"From the point of view of teaching and learning, it would not be easy to design a more difficult system than the English system. In contrast, it would seem almost impossible to design a system more easily learned than the metric system."[1]

The above quotation from the Twentieth Yearbook of the National Council of Teachers of Mathematics served as a motivation for this section.

The brief historical sketch is intended to help the pupils see the decimal foundation and origin of the metric system.

Although the pupils have had a brief introduction to the metric system in the 7th grade, we urge you to have them "live metric" for a few days. Postpone any attempts at converting from metric to English measures or vice versa. The pupils should have a metric ruler at hand in order to get well acquainted with the linear units. We suggest also that you use the metric system frequently in subsequent chapters in which measurements are required.

Because of the length of this chapter we were not able to introduce the centigrade scale for temperature measurements. You will also note that we have not developed in any detail the two systems now in vogue, namely, the MKS (meter, kilogram, second) or the CGS (centimeter, gram, second). As a matter of fact, we borrowed from both systems in order to make our tables more easily understandable.

We highly recommend the Twentieth Yearbook of the National Council of Teachers of Mathematics as a rich reference text for both your professional library and your pupils' mathematics library.

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[1] Clark, John R., "A Note on the Yearbook," The Metric System of Weights and Measures, Twentieth Yearbook of National Council of Teachers of Mathematics, Bureau of Publications, Teachers College, Columbia University, New York, 1948.

You will notice that without additional comment we have referred to the gram as the unit of mass, not weight, in the metric system. It was thought best not to involve the pupils in a full-scale discussion of this point here. An adequate treatment of the ideas belongs in a high school physics course. It is likely, however, that some of your pupils will ask questions about this terminology, and you will want to know how to answer them correctly, if not in complete detail. The following discussion should be adequate for this purpose. Should you wish more information on the subject, you might refer to Physics, Volume I, prepared by the Physical Science Study Committee of Educational Services, Inc. (This is the first volume of the so-called M.I.T. course for high school physics students.)

What is the weight of an object? It is a measurement of the force or "pull," of gravity on that object. An ordinary bathroom scale measures this pull by the amount it stretches, or twists, a spring. We think of weight as measuring the "quantity of matter" in an object, in some sense. A box of lead weighs more than the same box filled with feathers because the lead has a greater "quantity of matter" packed into the given volume than do feathers. There is another way to measure the "quantity of matter" of an object. This is to compare the object with some standard, or unit, bodies on a balance. If we have a supply of identical objects called "grams" we can determine the number of these "grams" it takes to balance the box of lead. This number of grams we call the mass of this much lead.

These two different ways of measuring "quantity of matter" can be used interchangeably for most purposes in any one fixed location, but they are not, strictly speaking, measurements of the same thing. Weight depends on the nearness to the center of the earth. The pull of the earth--gravity--on the box of lead would be much smaller in a space ship as far from the earth as is, say, the moon. The weight of the lead would be much smaller there. However, the lead would balance the same number of "grams" on the space ship which it balanced on the earth (the "grams" would themselves weigh correspondingly less) so its mass would be unchanged. To summarize:

> Weight is the pull of the earth. It changes as the distance between the object and the center of the earth changes.
>
> Mass is a comparison of the object with a set of unit bodies. It does not depend on the position in space where it is measured.

We humans are normally restricted to a very narrow range of altitude above sea level, and, with that restriction, we can think of weight and mass as having a definite fixed relationship. (It is tempting to predict that as we enter the space age and are released from these restrictions, weight and mass and the distinctions between them will become subjects for household discussion.) In the English System weight is measured in pounds, mass in slugs. An object which has a mass of 1 slug has a weight of approximately 32.2 pounds at sea level. The weight in pounds of any object is approximately 32.2 times its mass in slugs. The more common unit in this system, of course, is the pound. In the metric system, when mass is measured in grams, weight is measured in dynes. An object whose mass is one gram has a weight of approximately 980 dynes at sea level. The weight of this same object in the English System would be approximately 0.0022 pounds. As you probably know, the more familiar unit in the metric system is the unit of mass, the gram.

In the text we have introduced only the more familiar units, pound and gram. This has made it necessary to use both words, mass and weight. You must judge for yourself how much of the above discussion of the two ideas you will use in your classroom. If the subject does come up, however, be sure to make one point: both mass and weight can be measured in either of the systems of units, English and metric. If you fail to point this out to the pupil, he may interpret the discussion in the text to mean that weight is something measured in the English system and mass something measured in the metric system.

Answers to Exercises 3-7a. Pages 132,133.

1. (a) 1111.111 meters = 1 kilometer +
1 hectometer +
1 dekameter +
1 meter +
1 decimeter +
1 centimeter +
1 millimeter

or 1 km. + 1 hm. + 1 dkm. + 1m. + 1 dm. + 1 cm. + 1 mm.

(b) 5m. + 3dm. + 4 cm. + 2mm.

(c) 2 hm. + 4 dkm. + 5 m. + 3 cm. + 6 mm.

(d) 5 dm. + 6 cm. + 4 mm.

(e) 6 km. + 0 hm. + 4 dkm. + 3 m. + 2 dm. + 7 cm. + 8 mm.

(f) 2 km. + 0 hm. + 2 dkm. + 0 m. + 2 dm. + 0 cm. + 2 mm.

2. (a) 10
(b) 100
(c) 1,000
(d) 10,000
(e) 100,000
(f) 1,000,000

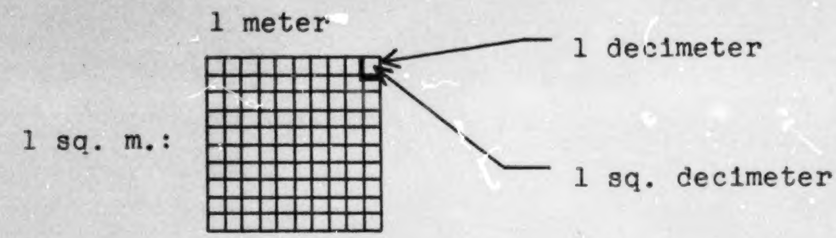
3. (a) 500
(b) 2000
(c) 0.5
(d) 25.4
(e) 1500
(f) 325
(g) 3.5
(h) 4.74
(i) 55
(j) 625

4. (a) 4×10^7
(b) 4×10^6
(c) 4×10^5

Answers to Exercises 3-7b. Page 134.

1. (a) 10,000
(b) 1,000,000
(c) $\frac{1}{100}$
(d) $\frac{1}{10,000}$
(e) $\frac{1}{1,000,000}$

2.



3. (a) 322 sq. cm.
   (b) 12.768 or 13 sq. m.
   (c) 3589 sq. cm. or 0.3589 sq. m.
   (d) 15 sq. mm. or 0.15 sq. cm.

4. 12.768 sq. m. = 127,680 sq. cm.

5. 113.04 sq. m.

6. 2.56 sq. m.

7. 29 cm.

Answers to Exercises 3-7c. Page 136.

1. (a) 1,000,000
   (b) 1,000,000,000
   (c) $\frac{1}{1000}$
   (d) $\frac{1}{1,000,000}$
   (e) $\frac{1}{1,000,000,000}$

2. 352.8 cu. cm.
3. 51.1 cu. cm. or 51,100 cu. mm.

Answers to Exercises 3-7d. Page 139.

1. (a) 4 kg. + 4 hgm. + 4 dkgm. + 4 gm. + 4 dgm. + 4 cgm. + 4 mgm.
   (b) 5 hgm. + 6 dkgm. + 7 gm. + 8 dgm.
2. (a) 352.8 gm. (b) 0.3528 kgm.
3. (a) 6 kl. + 6 hl. + 6 dkl. + 6 ℓ. + 6 dl. + 6 cl. + 6 ml.
   (b) 2 ℓ. + 0 dl. + 3 cl.
4. (a) 1 cc. = 1 ml.
   $\therefore$ 352.8 cc. = 352.8 ml.
   (b) 1 kgm. = 1 ℓ.
   $\therefore$ 0.3528 kgm. = 0.3528 ℓ

Answers to Exercises 3-7e. Page 140.

1. (a) 2.54 cm. = 1 inch; 10 × 2.54 = 25.4

   ∴ 25.4 cm. = 10 in (Since 2.54 cm. is defined to be equal to 1 inch, the equality sign is used.)

   (b) 1 meter $\approx$ 1.1 yards

   ∴ 100 m. $\approx$ 100 × 1.1 $\approx$ 110 yd.

   (c) 200 m. $\approx$ 220 yd.

   (d) 400 m. $\approx$ 440 yd.

   (e) 800 m. $\approx$ 880 yd.

   (f) 1500 m. $\approx$ 1650 yd.

   (g) 1 km. $\approx$ 0.6 mi.; ∴ 10 km. $\approx$ 10 × 0.6 or 6 mi.

   (h) 100 km. $\approx$ 60 mi.

   (i) 1 ℓ. $\approx$ 1 qt.; ∴ 4 ℓ. $\approx$ 4 qt. $\approx$ 1 gal.

   (j) 60 ℓ., 60 qt., 15 gal.

2. (No allowance is made for the approximate nature of measures.)

   (a) 81 × 81 × 81 = 531,441 (cu. in.)

   (b) $\frac{531,441}{1728}$ = 307.5 (cu. ft.)

   (c) 307.5 × 62.4 = 19,188 (lb.)

3. (a) $(2.06)^3 = 8.741816$ (cu. m.)

   (b) 8,741.816 liters

   (c) 8,741.816 kilograms

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4. Problem 3 should take less time because only one set of computations needs to be made.

   Also, one more measure (liters) is found.

To convert a number of cu. in. to gallons, a further computation would have to be made. There are 231 cu. in. in a gallon.

$$\frac{531,441}{231} \approx 2300.6 \quad \text{(gallons)}$$

Research Problem:

(c) Gold is weighed in troy units, while feathers are weighed in avoirdupois units. The pound of feathers weighs more.

Sample Questions

1. TRUE-FALSE
   (a) An exponent tells us by what number the base is multiplied.
   (b) $10^2 \cdot 10 = 10^2$
   (c) $10^5 \cdot 10^{-5} = 1$
   (d) $1,000,000 = 10^6$
   (e) $1\frac{1}{2}\% = 1.5 \times 10^{-2}$
   (f) $\frac{1}{10^5} = \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$
   (g) 2.76 billion = 2.76×10^9
   (h) $10^3 = 10^{-2} + 10^5$
   (i) $93,000,000 = 9.3 \times 10^6$
   (j) $10^3 \cdot 10^3 = 100^6$
   (k) 11×10^7 is written in scientific notation.
2. Write the following using scientific notation.
   (a) 0.04
   (b) 5680
   (c) 61.3207
   (d) 28 million
   (e) 0.00495×10^6
   (f) 0.2×10^{-3}
3. Perform the indicated operations - Write answers in scientific notation.
   (a) $(5 \times 10^{-6}) \times (10^{12})$

(b) $(6 \times 10^{-5}) \times (3 \times 10^{-3})$
(c) $19.6 \div 0.028$
(d) $0.787 \times 200{,}000$

4. Fill in the blanks with the proper symbol.
   (a) $7.81 = 781 \times 10^{\square}$
   (b) $61320 = 613.20 \times 10^{\square}$
   (c) $1\% = 1 \times 10^{\square}$
   (d) $7.60 \times 10^{3} = 7.600 \times 10^{\square}$

5. Place the proper symbol, × or ÷, in the ☐ to make the following true.
   (a) 100 mm. = 100 m. ☐ 10^{3}
   (b) 1 cm. = 1 m. ☐ 10^{2}
   (c) 3 km. = 30 m. ☐ 10^{2}
   (d) 63 m. = 6.3 cm. ☐ 10^{3}
   (e) 1 m. = 1 mm. ☐ 10^{3}

6. To find 10% of a number one may multiply the number by the ___?___ power of 10.

7. To change from a certain standard unit (in the metric system) of measure to a larger one it is necessary to ____?____ (multiply or divide?) by a positive power of 10.

8. Write in words the value of 6143 after it has been rounded to the nearest ten.

9. Express $\left(\frac{1}{1000}\right)^{3}$ in scientific notation.

10. To divide 10^{a} by 10^{b} where a and b are integers, it is necessary only to __?__ b from a and use this result as an exponent of 10.

Answers to Sample Questions

1. (a) False
   (b) False
   (c) True
   (d) True
   (e) True
   (f) True
   (g) True
   (h) False
   (i) False
   (j) False
   (k) False

2. (a) 4×10^{-2}
   (b) 5.68×10^{3} or 5.680×10^{3}
   (c) 6.13207×10
   (d) 2.8×10^{7}
   (e) 4.95×10^{3}
   (f) 2×10^{-4}

3. (a) 5×10^{6}
   (b) 1.8×10^{-7}
   (c) 7×10^{2}
   (d) 3.935×10^{-6}

4. (a) $^{-}2$
   (b) 2
   (c) $^{-}2$
   (d) 3

5. (a) ÷
   (b) ÷
   (c) ×
   (d) ×
   (e) ×

6. $^{-}1$

7. Divide

8. Six thousand one hundred forty

9. 10^{-9}

10. Subtract

Chapter 4

DRAWINGS AND CONSTRUCTIONS

This chapter should require about 10 days. Sections 1, 2, 5, and 6 should take one day each. Sections 3, 4, and 7 will probably require two days each.

There are several reasons for introducing a chapter on geometric drawings at this time. Two of these are pointed out in the student's text: (1) a drawing or diagram helps students visualize the relations in problems, and (2) some problems can be solved by using accurate drawings. In addition to these reasons, teachers should be aware of constructions in geometry which will be studied in high school. Students who are familiar with basic constructions will be able to concentrate on the validity of the constructions and make use of the basic principles without loss of time needed to develop these skills. Finally, this type of work is usually enjoyed by the junior high school student, particularly the section on geometric designs.

This chapter deals with two types of geometric drawings together with an elementary sampling of some types of problems that make use of such figures. Section 2 deals with methods of drawing accurate plane figures with the aid of a ruler and protractor. Sections 3, 4, and 5 present the basic classical constructions using only a compass and straightedge together with some construction problems and geometric designs. Section 6 makes use of both types of drawing tools in the construction of scale drawings. Section 7 is more difficult; the students are introduced to projection in drawing representations of figures in space.

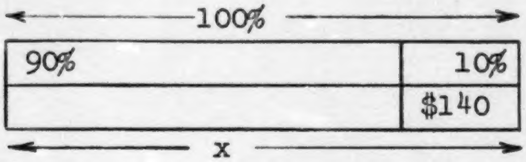
The term "construction," as used in this chapter, is limited to the classical constructions but note is made that this is to simplify the awkward phrase "compass and straightedge construction." In later work students will be given constructions using ruler and protractor as well as other aids in making accurate figures.

Mention is made of some of the tools used by draftsmen but, since it is felt that few students or schools will have such tools available, no work with these tools is introduced.

There is no attempt at any point in the chapter to prove the accuracy of any of the constructions given. This is left to the more formal geometric work of high school. A few of the brighter students may be able to prove the basic constructions after they have the work on congruence in a future chapter.

4-1. Introduction to Mathematical Drawings and Constructions. Pages 144-146. The exercises for this section are general. The first question in the set gives the students an opportunity to find out more about the tools used by draftsmen. A secondary dictionary gives a brief description of these tools, and an unabridged dictionary gives a better explanation. Students should be encouraged to find more complete descriptions. French curves are now available in geometry tools that can be purchased in most ten cent stores and 30-60 and 45-45 triangles are readily available. Some might be brought to class by some students. A pantograph is a relatively simple tool; some of the boys (or girls) might be interested in trying to construct one. Webster's unabridged dictionary gives an explanation that should be adequate for this purpose. The rest of these exercises provide an opportunity to strengthen problem-solving skills. A picture or diagram is an excellent device to help students visualize relationships. Some of these problems are so easy that the better student will not need any diagram, but developing the habit of making rough sketches is to be encouraged; it eliminates many useless questions and much careless thinking.

Some students may not think of diagramming per cent relations. Many students can be helped to understand these relationships when a rectangular bar is used to represent the whole quantity. If the bar is split lengthwise, the per cent relation can be pictured in one half and the corresponding relation in money or any other item in the other half. When these relationships are visualized students have little difficulty in setting up a proportion or equation to solve the problem. As an example of this, consider this problem: a theater manager banks 90% of his gross receipts and pays the rest to the Federal Government for entertainment tax. If he pays a tax of $140, what were his gross receipts? This type of problem presents difficulties since the money given is not the 90% mentioned in the problem, and the answer called for is the entire 100% given in money.



As a proportion this can be set up as:

$$\frac{10}{100} = \frac{\$140}{x}$$

Then, $10x = \$14000$

and $x = \$1400$

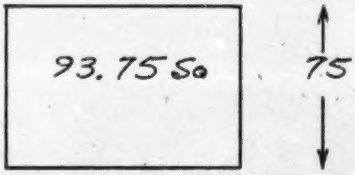
As an equation, this becomes

$0.10x = \$140$

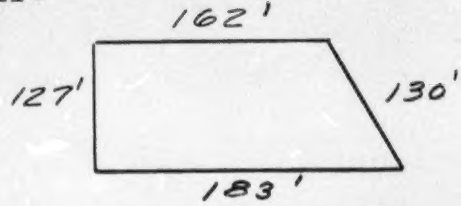
and $x = \$1400$

Answers 4-1. Pages 144-146.

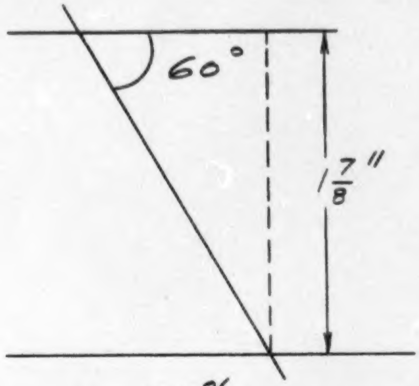
1. (a) A right triangle that has one acute angle with a measure of 30° and the other of 60°.
   (b) A set of stencils used to draw various curves. Some inexpensive curves are now available in plastic, often combined with a ruler, protractor and other geometric tools.
   (c) A machine used for changing the size of a figure accurately.
2. Rough sketch
   125 feet



3. Rough sketch
602 feet



4. Accurate drawing
346 feet



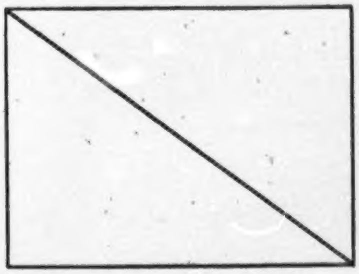
5. Rough sketch
$22.40

100%

80%	20%
$ 17.92	

$ x

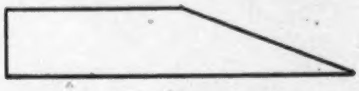
6. Accurate drawing
≈15 feet



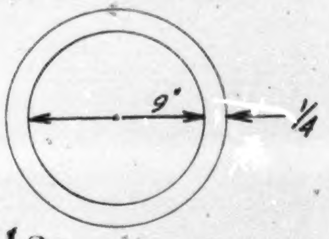
7. Rough sketch
$157.50

55%	45%
$192.50	$ x

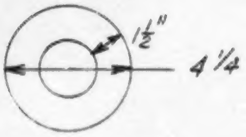
8. Accurate drawing
≈1 $\frac{1}{16}$ feet



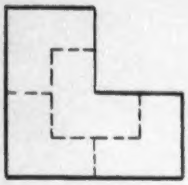
9. Rough sketch
≈9 $\frac{1}{2}$ "



10. Rough sketch
$\approx 1\frac{1}{4}$"



11.

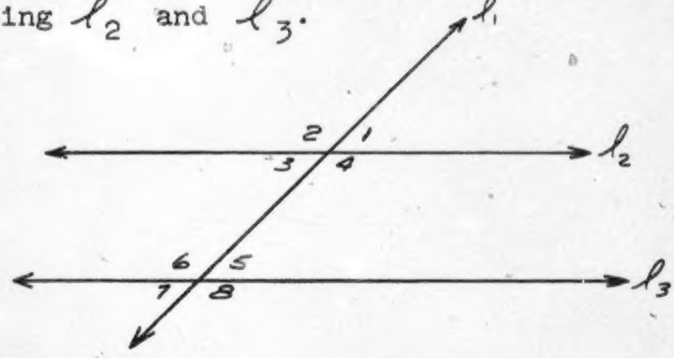


4-2. Protractor and Ruler Drawings. Pages 146-150

In this section the pupils are to practice using the protractor and the ruler. Careful use of these tools should be stressed. Although tools other than these two are mentioned, and may be used if available, the main purpose of the section is to learn to draw figures accurately using these tools.

There are two expressions which the pupils have had before that they may have forgotten. They are: transversal and corresponding angles. It may be necessary to review these before the pupils use them.

(1) Transversal. A transversal is a line that cuts two or more lines. In the figure, ℓ_1 is a transversal cutting ℓ_2 and ℓ_3.



(2) Corresponding Angles. When a transversal cuts two lines, as in the figure above, eight angles are formed.

This gives four sets of corresponding angles containing two angles to the set. Angles are said to be corresponding when they lie on the same side of the transversal and on the same relative side of the lines. In the figure we have the following sets of corresponding angles.

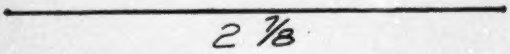
$\angle 4$ and $\angle 8$
$\angle 1$ and $\angle 5$
$\angle 3$ and $\angle 7$
$\angle 2$ and $\angle 6$

Answers to Questions in the Text.

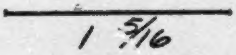
1. It is better not to use the zero point on most rulers because it decreases the chance for accuracy. In fact it is better not to use either end point of the ruler. On some rulers the corners may be broken making it difficult to determine the endpoints.
2. When drawing parallel lines it is not necessary for the second line to cross the first one, so as to form angles of 90°.

Answers to Exercises 4-2. Pages 150-151.

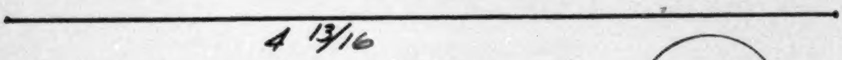
1. (a)



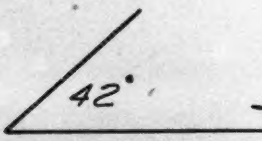
(b)



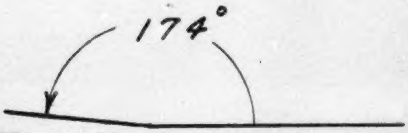
(c)



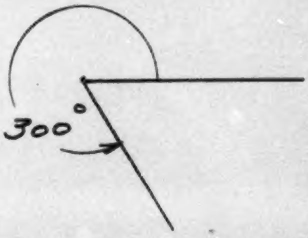
2. (a)



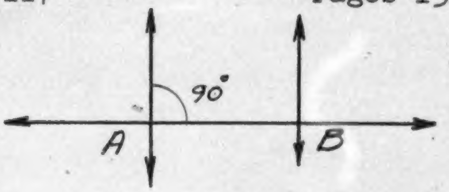
(b)



(c)

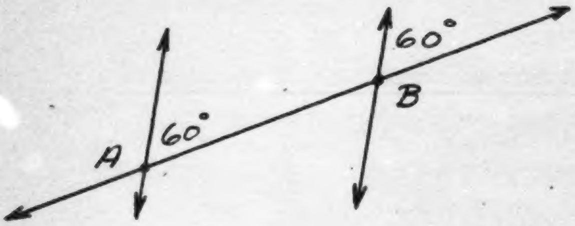


3. (a)

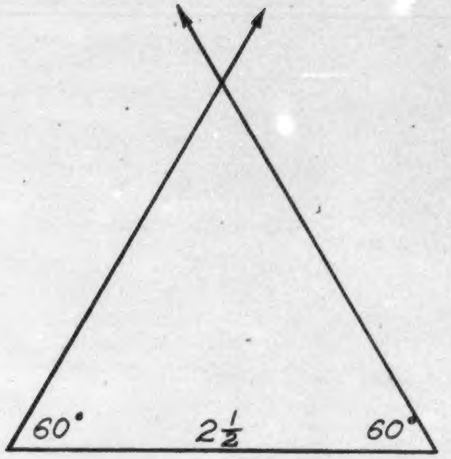


(b) Parallel

4.

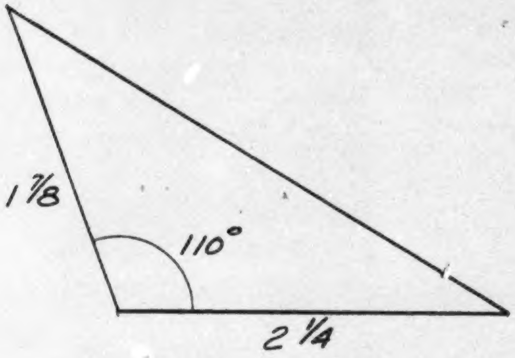


5. (a)

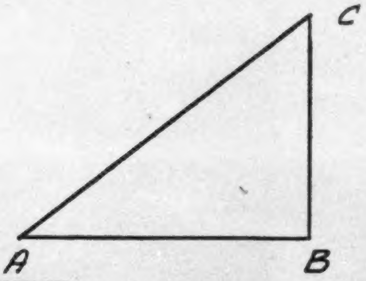

60°
$2\frac{1}{2}$
60°

(b) Equilateral

6.


$1\frac{7}{8}$
110°
$2\frac{1}{4}$

7. (a)



(b) Right triangle.

(c) 90°.

4-3. Basic Constructions. Pages 151-154.

This section deals entirely with basic classical geometric constructions. Students will need instruction on the correct usage of compasses. The compass should be held so as to keep the radius constant. This is particularly true since most students will be using inexpensive compasses that do not lock in position and there is a tendency for the radius to slip. Most of the inexpensive compasses used by students today have an inch scale; it is worthwhile to point out that this is not accurate since the position of the pencil point can be raised or lowered with a resulting change in radius. If a radius is to be some measure given in terms of a standard unit, it should be checked against a ruler. This is probably the first time that many students have thought of a compass as an instruement for measuring lengths. It should be stressed that a compass will measure equal lengths but will not measure in terms of inches. Of course it is possible to use the ruler to draw a segment with the length of a standard unit of measure and then copy this segment using the compass.

Six basic constructions are given. The first, measuring equal segments, is usually assumed, but since this is a new concept for students of this age, the construction is given in detail. The constructions for bisections, perpendiculars and copying an angle do not deviate from the traditional; the vocabulary is consistent with modern terminology.

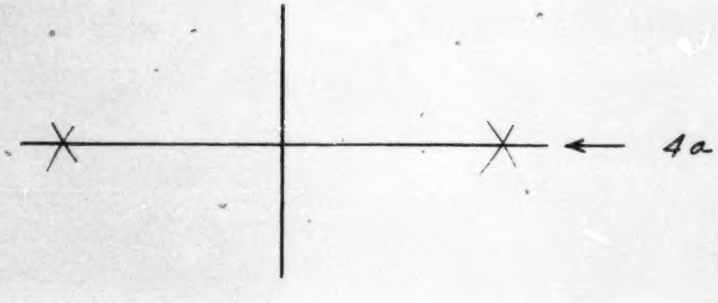
When it is possible to present a construction easily without lettering the drawing, this is done. This permits a more general description of the construction. At times the English statement becomes so cumbersome that lettered drawings are used to simplify the statement. It should be pointed out to students that either of two intersections can be used first as the center of an arc when directions specify one lettered point. Lettered figures are used to explain the bisection of an angle and one of the perpendicular constructions.

Some problems in this section are leading up to the concept of concurrent lines. In exercises 4-3a, Problem 7, the students are asked to construct the perpendicular bisectors of each side of a triangle and extend the lines until they intersect. It is possible that some students will draw triangles in such a way that the intersection is off the page. At this stage the student should probably be encouraged to draw either a smaller triangle or a triangle of different shape, and try again. The teacher should enlarge upon the situation in class to help students see that eventually these lines will intersect. A similar situation may arise in exercises 4-3b, Problem 5.

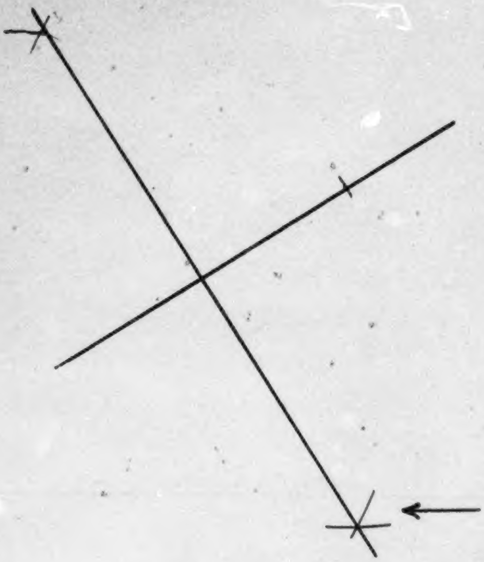
The relationship between the construction for bisecting a line segment and the erection of perpendiculars is brought out in exercises. It is hoped that the students will be able to discover the similarity for themselves. If necessary, a few leading questions might be used to help show that, in perpendiculars, a segment is chosen so that its endpoints are equidistant from the point through which the perpendicular passes. From this step, the method used for constructing the perpendicular bisector is used.

Answers. 4-3a. Page 155.

1.

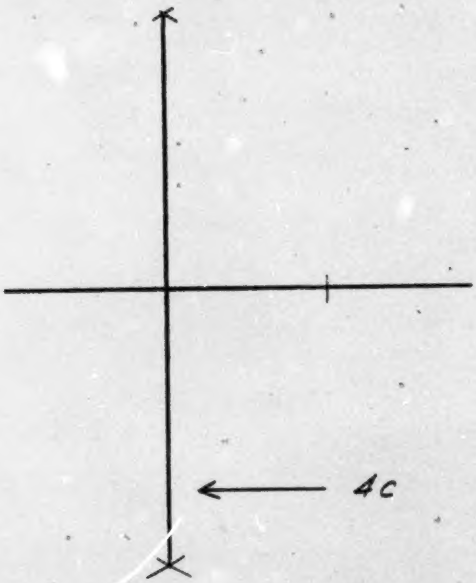


2.



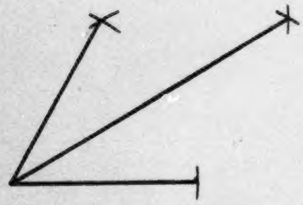
4 b

3.

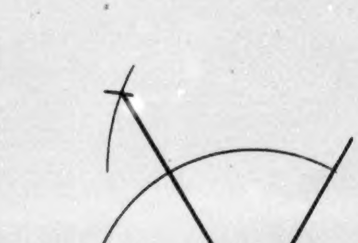


4c

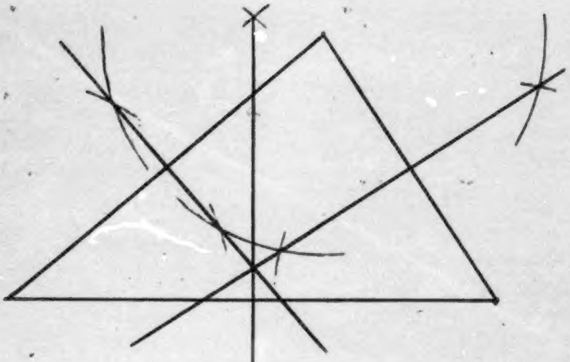
5.



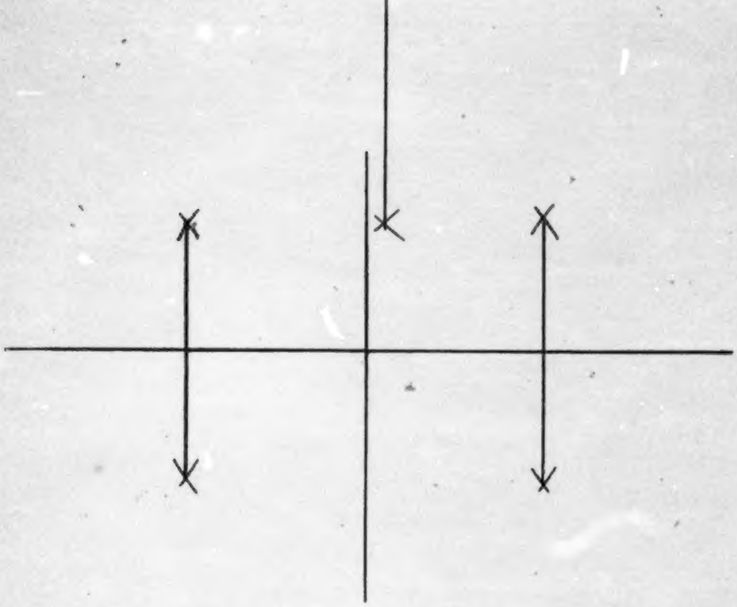
6.



7.

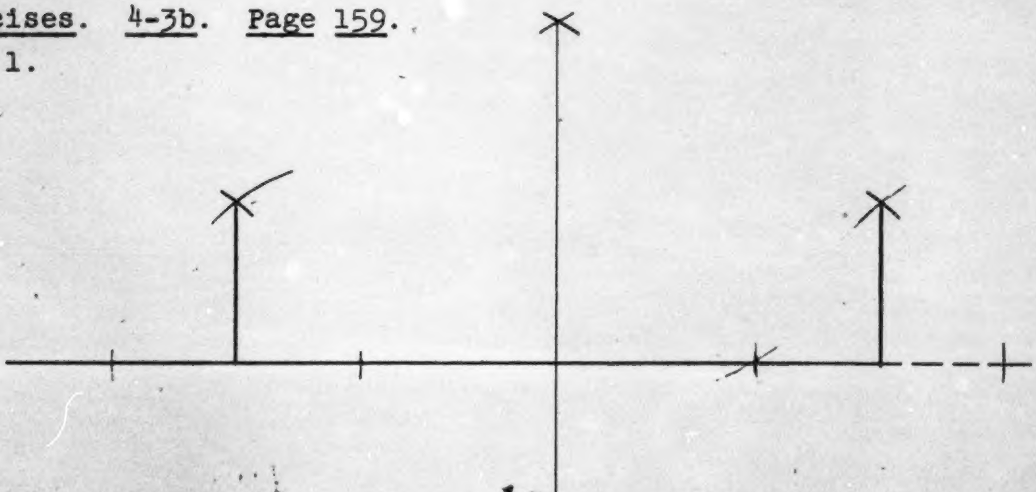


8.

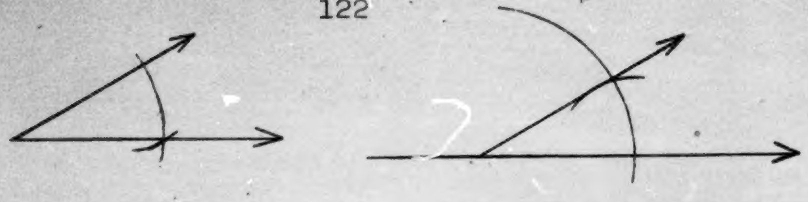


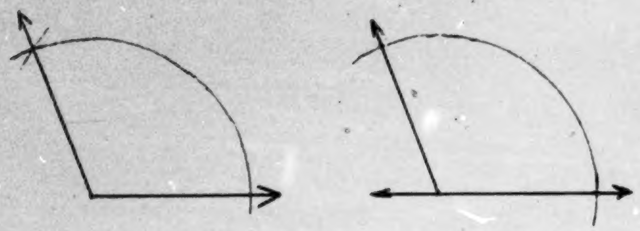
Exercises. 4-3b. Page 159.

1.

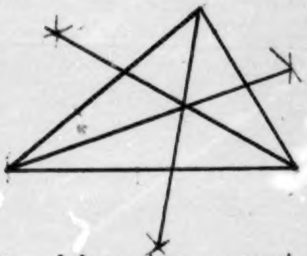


2.



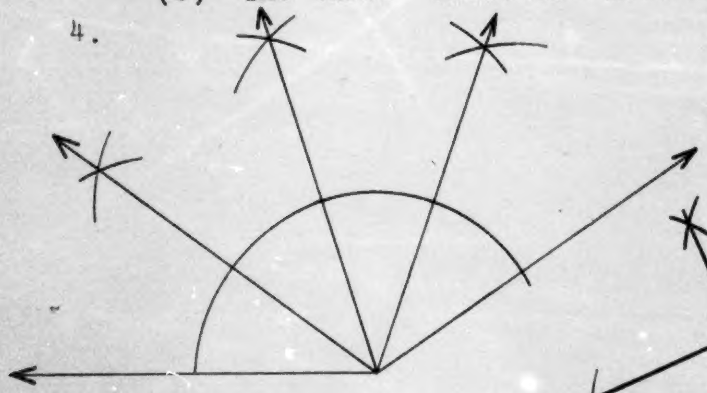


3. (a)

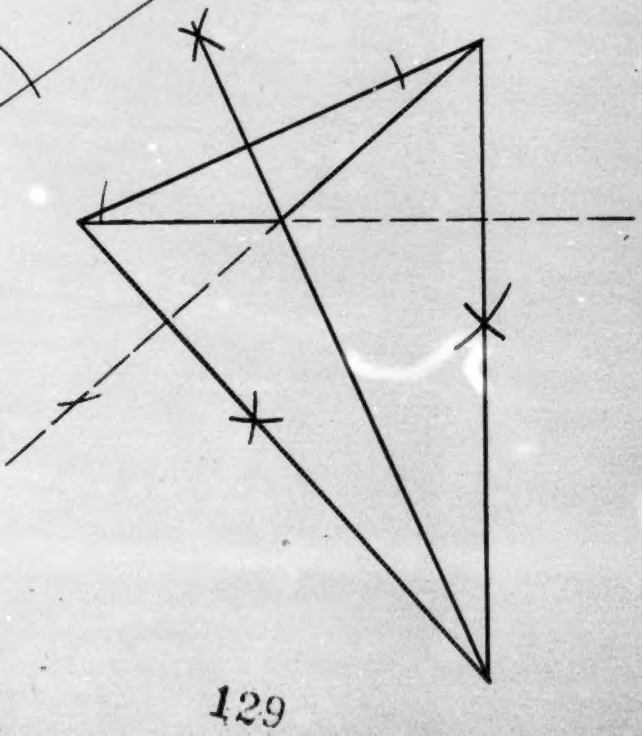


(b) The three bisectors meet in a point.

4.

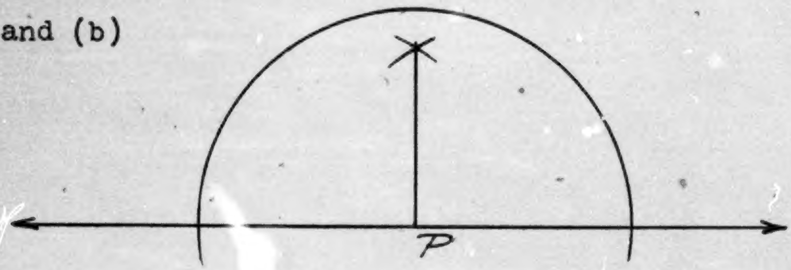


5. (a)



(b) The perpendiculars meet in a point.

6. (a) and (b)



(c) Both erect the perpendicular through the midpoint of a segment.

(d) In erecting a perpendicular it is necessary to locate the segment by marking arc equal distances from the point. Only one additional point is needed for perpendiculars while it is necessary to find two points when bisecting a segment.

*7. (a) Use the point as center and mark two arcs that intersect the line at 2 points. Call these points A and B. A and B are at equal distances from the point.

(b) Set the compass so that the radius is more than half the distance AB.

(c) Use point A as center and draw an arc above or below the central part of the segment.

(d) Repeat with B as center.

(e) Connect the intersections of the last two arcs to the original point.

4-4. Constructions Problems. Page 160.

Section 4 makes use of the basic constructions taught in Section 3 in a series of problems. This set is rather long and probably should be given as two assignments.

Without putting too much stress on the topic an attempt is made to let the student develop constructions for triangles to match the three basic conditions of congruency.

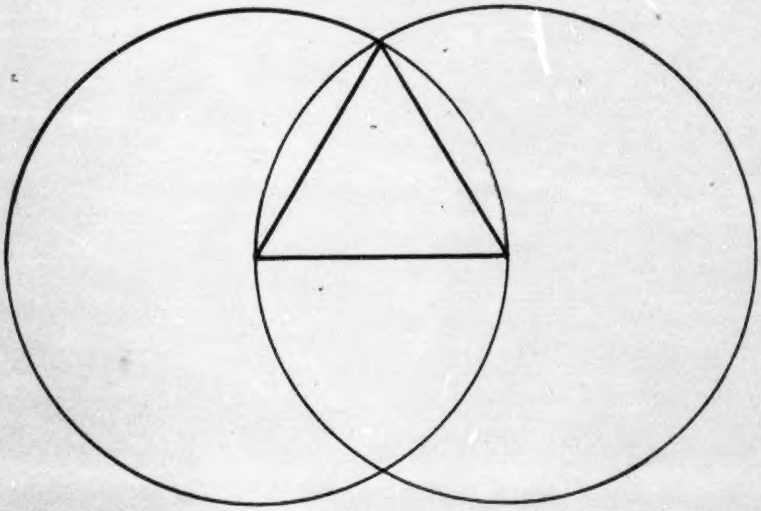
No mention of congruence is made at this time, but, as a prelude to congruence, the student is asked whether all triangles with three measurements the same will look alike. Students probably will need help to see that triangles that are rotated or are mirror images of each other are alike.

Some teachers may wish to do more with these constructions in Chapter 5 when the topic of congruence will be studied. This set of exercises is probably one of the most difficult in the chapter. It is hoped that the better students will be able to develop these concepts for themselves with the hints given with these problems. If it proves too difficult, the teacher may want to enlarge upon the problems.

The concept of concurrent lines is used in connection with the interesting sets of concurrent lines found in triangles. The concept of concurrent lines is simple and should cause no problem; the better students may be sufficiently intrigued to do some independent work with this. The brainbuster, problem number 14, starts them on this trail.

Exercises. 4-4. Pages 160-163

1. a, b, and d
   c. 2



(e) The three segments have the same length.

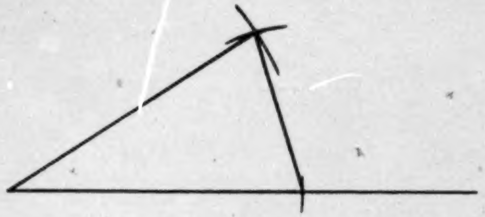
(f) Equilateral triangle.

(g) No. Arcs that intersect are all that are needed.

(h) Each angle has a measurement of 60°.

(i) Segments are the same because they are radii of the same circle.

2.



*3. (a) Draw a base line and mark off a segment equal to one side of the triangle.

(b) Set the radius of the compass so that it is equal in length to the second segment.

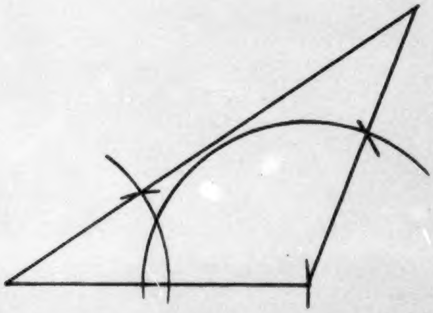
(c) Place the point of the compass at one endpoint of the first segment and draw an arc on one side of the line.

(d) Set the radius of the compass so that it is equal in length to the third segment.

(e) Place the compass at the other endpoint of the first segment and draw an arc that intersects the arc drawn in step a.

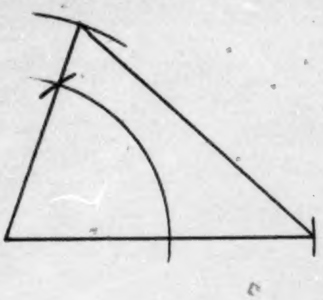
(f) Connect the intersection of the arcs to each endpoint.

4. (a)



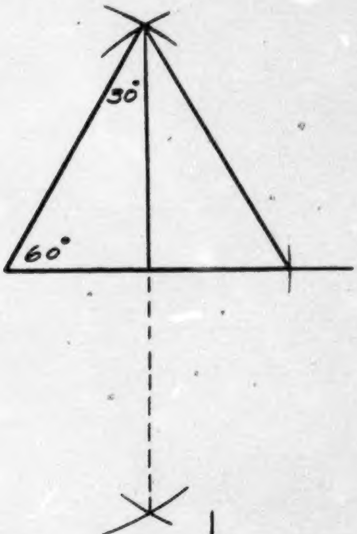
(b) Yes, but one might be a mirror image of the other.

5. (a)



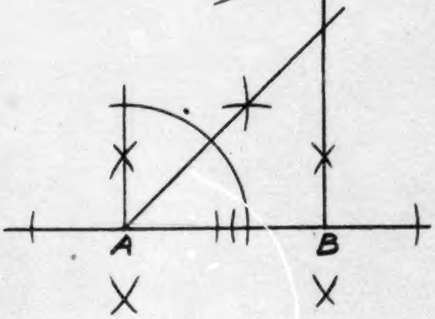
(b) Yes, but one might be a mirror image of the other.

6. (a)



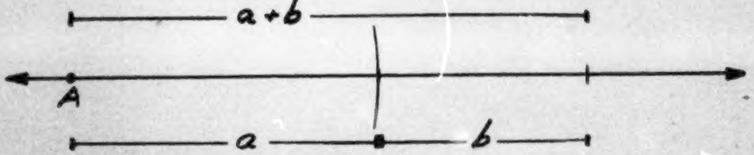
(b) 30°

7. (a)

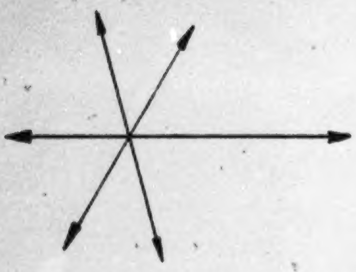


(b) 45°

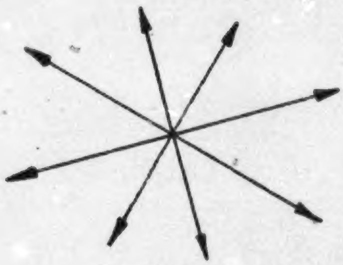
8.



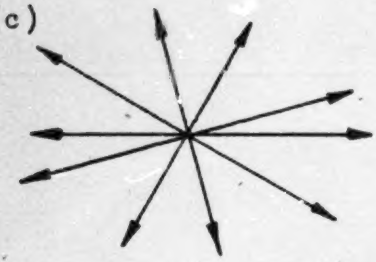
9. (a)



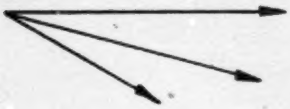
(b)



(c)

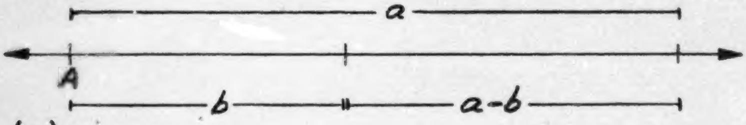


10.

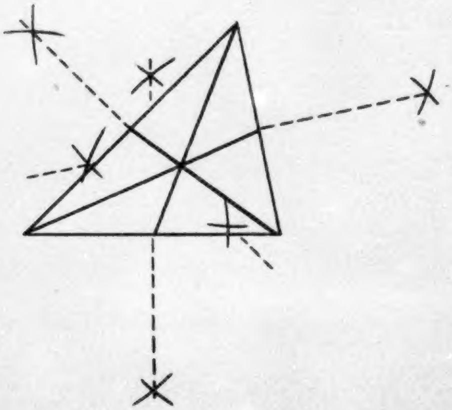


11. 3, the two small angles and the large angle. Using exterior angles or equally spaced rays 6 angles are possible

12.

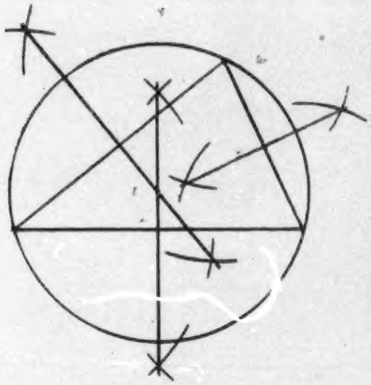


13. (a)



(b) yes

14. Brainbuster:



(a) The perpendicular bisectors of each side.

(b) Every point on a perpendicular bisector is the same distance from the endpoints of the segment bisected. A circle can be drawn through the endpoints of each segment from any point on the bisector. The point where the three bisectors meet is the same distance from all endpoints or vertices. This distance is the radius of the circumscribed circle.

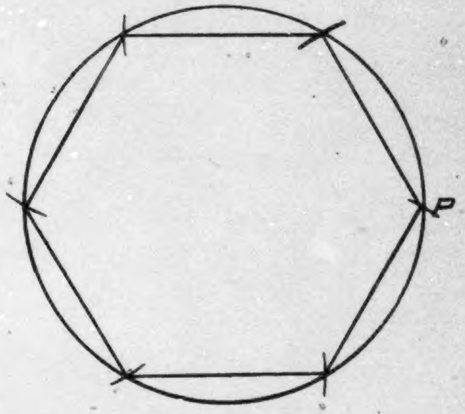
4-5. Inscribed Figures. Page 163.

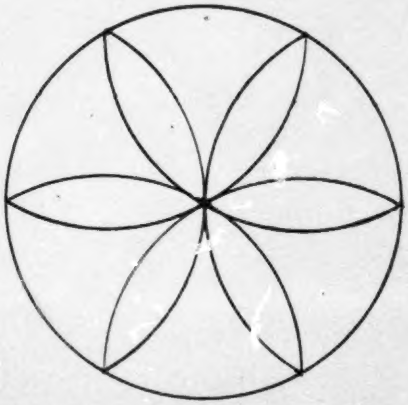
Methods for inscribing triangles, squares, hexagons, and octagons are developed in Section 5. With these basic constructions and ideas, students can copy geometric designs and can create their own. This is a topic in mathematics where many students who have not been outstanding previously, have an opportunity to shine. Into every life, some joy should fall.

Students who have had SMSG work in seventh grade are familiar with the term circle as a set of points that are equal in distance from a central point and should be reminded of the definition. Since this definition eliminates the interior (or area) of the circle as part of the circle, students who have had other work should learn to consider the term "circle" to be the same as "the circumference of the circle." After the basic processes are learned, most eighth grade students enjoy creating their own designs. This need not take much class time since most of this can be done outside of class.

Exercises. 4-5. Pages 163-165.

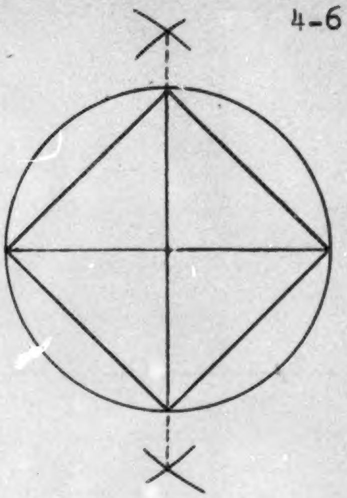
1. (a) 6
   (c) Hexagon
   (d) Connect every other intersection.
   (e) Connect every other intersection and then connect those omitted at first.
   (f) Six petal flower





2. (a) Square

(b) Bisect each angle and extend the rays to the circle forming 8 diameters. Connect the endpoints in order. An alternate method is to bisect each side and draw the bisectors so that they intersect the circle. Again connect the 8 points of intersection in order.



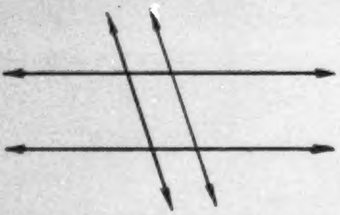
4-6. Scale Drawings. Pages 166-167.

The material of this section is important and very useful. Again, the pupils need to use extreme care in making the drawings. There should be a discussion on the selection of the proper unit to be used in scale drawings, and also a discussion on changing the measures of the object to the corresponding measures of the unit scale selected.

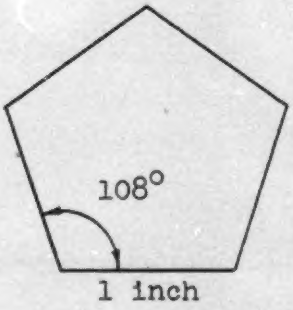
In the text the question is asked: "Where did $\frac{160}{3}$ come from?" A football field is $53\frac{1}{3}$ yards wide. This is equal to 160 feet. In working the proportion it is easier to use the form $\frac{160}{3}$ yards rather than $53\frac{1}{3}$ yards.

Answers to Exercises 4-6. Page 168.

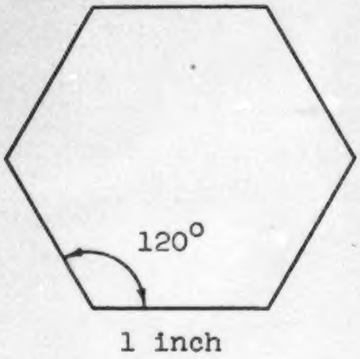
1.



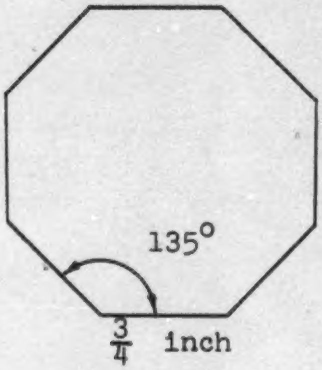
2.



3.



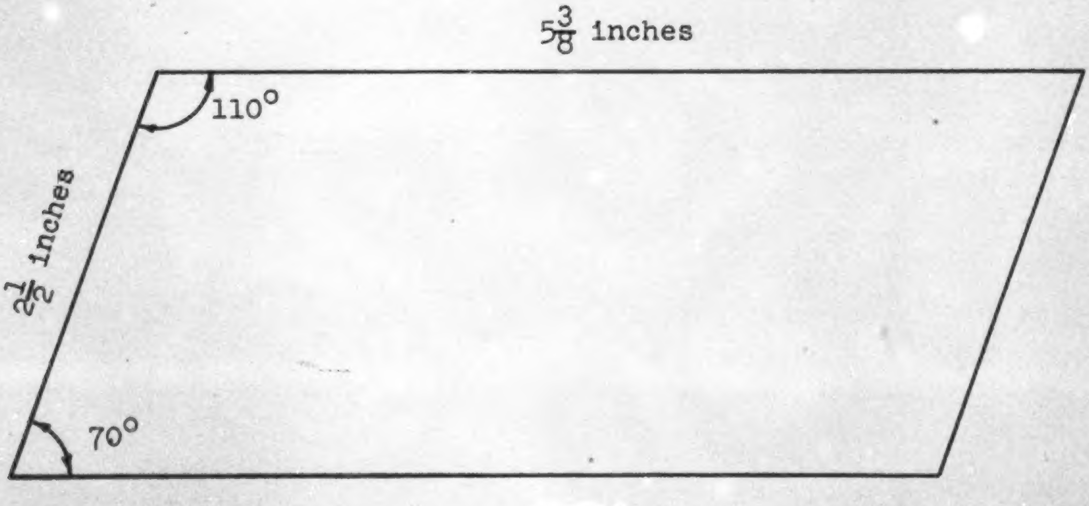
4.


135°
$\frac{3}{4}$ inch

5. 850 miles.

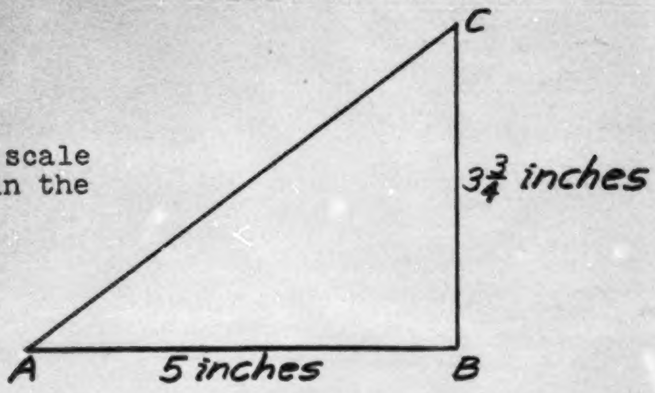
6. $3\frac{3}{4}$ inches.

7.


$5\frac{3}{8}$ inches
110°
$2\frac{1}{2}$ inches
70°

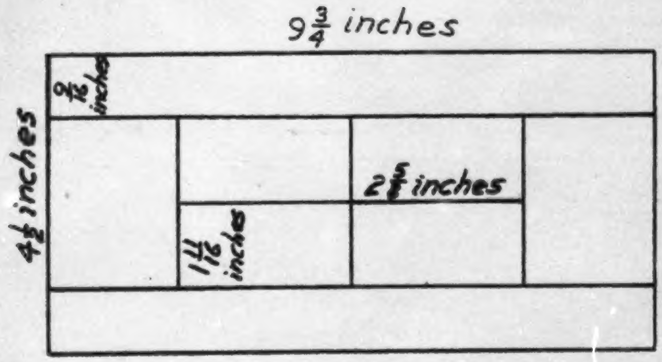
8.

(Not to scale given in the text.)


C
$3\frac{3}{4}$ inches
A
5 inches
B

The measurement of $\overline{AC}$ is $6\frac{1}{4}$ inches which represents 50 miles.

9.


$9\frac{3}{4}$ inches
$\frac{9}{16}$ inches
$4\frac{1}{2}$ inches
$2\frac{5}{8}$ inches
$1\frac{11}{16}$ inches

(Not to scale given in the text.)

4-7. Solids.

This section is probably the most difficult part of the chapter. For this reason adequate time should be devoted to class discussion of the text and especially of the text illustrations. Words will be used which the pupils have had before, such as prism and cylinder, but these concepts will need to be reviewed. There will be some new concepts, such as pyramid and cone. No effort is made to describe these solids completely in this chapter, since they will be studied in more detail later. Enough of a description is given, however, to enable the pupils to proceed with making the drawings.

Perhaps the most difficult concept is that of projection. Pupils who have had an art course may understand this concept. All members of the class can be helped through a proper discussion of this topic.

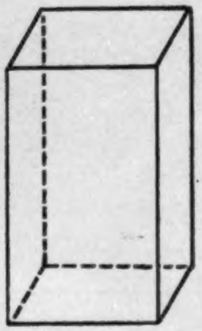
Drawing pictures of 3-dimensional objects on 2-dimensional space demands some distortion. Reality can be gained by use of perspective or projection. The drawings used in this section make use of dimetric projection. Students may be helped if this is compared to flat maps which use some projection, usually the Mercator. The word perspective is used loosely to indicate a distortion that appears to give depth. Getting the proper "perspective" is the crux of this section. Of course practice is essential in obtaining the desired results.

Answers to Questions in the Text.

1. Triangular prisms: (f) 5 faces (g) two are triangles and three
2. Hexagonal Prisms (d) 8 faces.
   (e) 2 faces are hexagonal and 6 are rectangular.
3. Pyramids. (e) 4 faces. The base is considered a face.
4. Intersecting planes: (d) Parallelogram

Answers to Exercises 4-7.

1.



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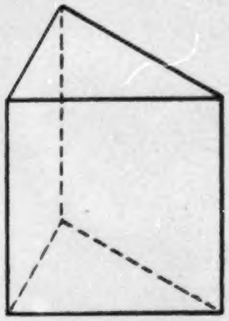

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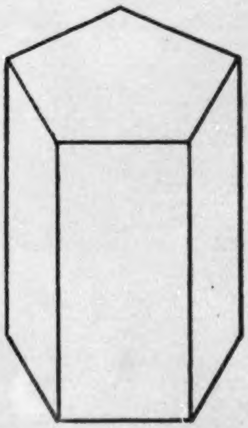

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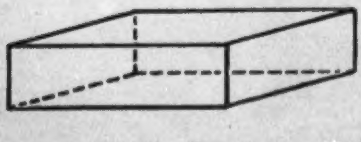

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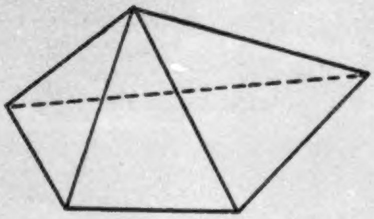

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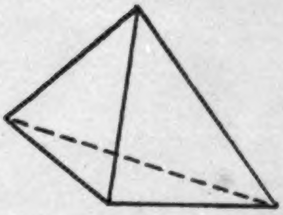

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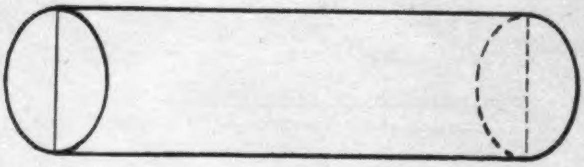
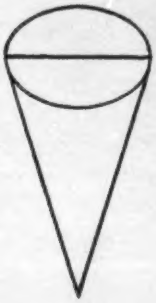


8.



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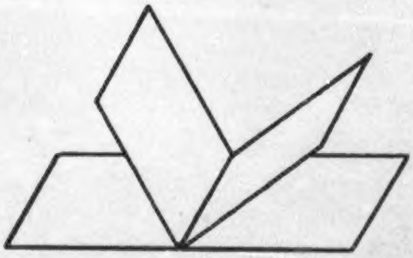
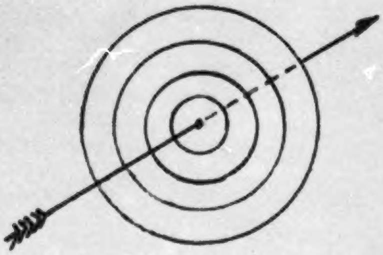


10.



Sample Questions, Chapter 4

Choose questions from this list. It is too long to include all questions in one test.

I. Use a ruler and protractor to draw each of these figures.

1. Draw a triangle that has a segment 3 inches long between angles of 35° and 65°.
2. Draw a triangle that has an angle of 100°. The segments on each side of this angle are $2\frac{1}{2}$ inches and $3\frac{1}{4}$ inches long.
3. Draw an equilateral triangle whose sides are each $1\frac{1}{2}$ inches long.
4. Draw an isosceles triangle whose base is $1\frac{1}{2}$ inches and whose equal angles are each 40°.
5. Draw a regular pentagon with a side $\frac{1}{2}$ inch long. Each angle in a pentagon is 108°.

*6. The sum of all seven angles in a heptagon is 900°. (A heptagon is a seven-sided polygon.) Find the size of each angle in a regular heptagon, draw a heptagon with each side $\frac{1}{2}$ inch long.

II. Use any tools available to make these scale drawings.

1. A softball diamond is really a square. The bases are 60 feet apart. Place the pitchers' mound at the intersection of the two diagonals. Draw the infield of a softball diamond. How far is it from the pitchers' mound to home plate?
2. Make a scale drawing of the end view of a garage that fits this description. The lower part is a rectangle 20 feet wide and 8 feet high. The roof forms an isoceles triangle and the distance from the peak to the walls is $12\frac{1}{2}$ feet. Center a door that is 18 feet wide and 7 feet high on this endwall.
3. Make a scale drawing of the floor plan of a gymnasium which is 60 feet long and 40 feet wide. Place three volleyball courts across the gym so that there is a 3-foot strip behind the serving lines and at each end of the gymnasium and twelve inches between each of the courts.

III. Use only compass and straightedge to make these constructions.

1. Construct a square with each side 2 inches long. (Use a ruler to measure this segment but use a compass to copy the sides from the segment.)
2. Construct an equilateral triangle.
3. Inscribe an equilateral triangle in a circle.
4. Construct a triangle whose sides are 2 inches, 3 inches, and 4 inches.

5. Why is it impossible to construct a triangle with sides of 1 inch, 2 inches and 3 inches? Show this by a construction.
6. Construct a design based on an inscribed hexagon.
7. Construct a design based on an inscribed octagon.
8. Construct a triangle with angles of 30°, 60°, and 90°.

IV. Use projection to draw pictures of these solid figures.

1. Draw a picture of a drum.
2. Draw a picture of a block that is in the shape of a cube.
3. Draw a triangular prism.
4. Indian tepees may have the shape of pyramids or cones. Draw a tepee of each type.
5. A monument has a square prism for the base topped by a square pyramid. Draw a picture of such a monument.
6. Draw a picture of a hexagonal prism.

V. Problems.

1. Basketball courts, in the form of a rectangle, vary in size. The largest official court is 94 feet long and 50 feet wide. Make a scale drawing and find the area of a basketball court of this size.
2. The scale of a map is $\frac{3}{4}$ inch = 100 miles. How many miles would be represented by a length of $4\frac{1}{2}$ inches on the map?
3. The distance between two cities is 1250 miles. If $\frac{1}{2}$ inch represents 150 miles, how long a segment would represent the distance between the cities?

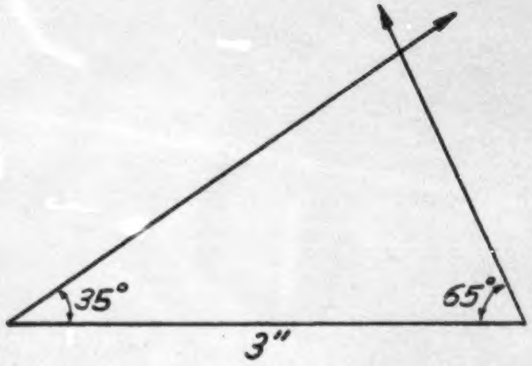
4. A plot of ground is in the shape of a parallelogram. The lengths of the sides are 54 yards and 27 yards. The acute angles are 70°. Make a scale drawing of this plot of land. Find its perimeter and area. (You will have to find the height of the parallelogram from the scale drawing.)

5. A running track has two parallel straightaways, each 220 yards long. They are 350 feet apart. The ends of these straightaways are connected by semi-circles. Draw a figure, then find the distance around this running track. How does this distance compare with 1 mile? (Use $\pi = \frac{22}{7}$)

Answers to Sample Questions.

I.

1.



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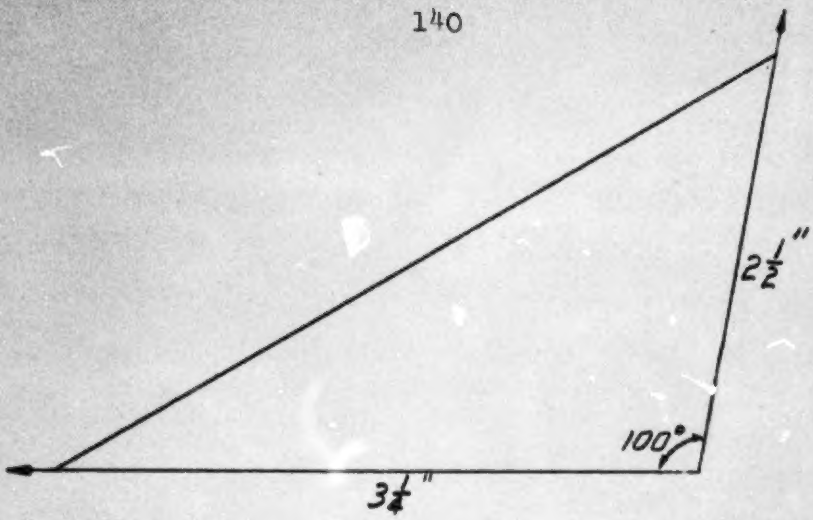
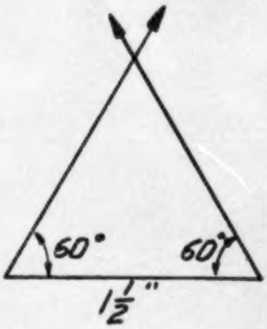
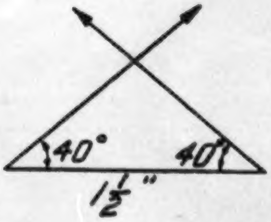


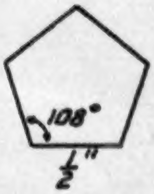
3.



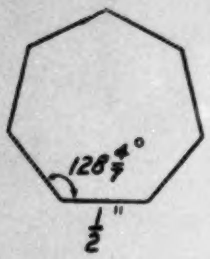
4.



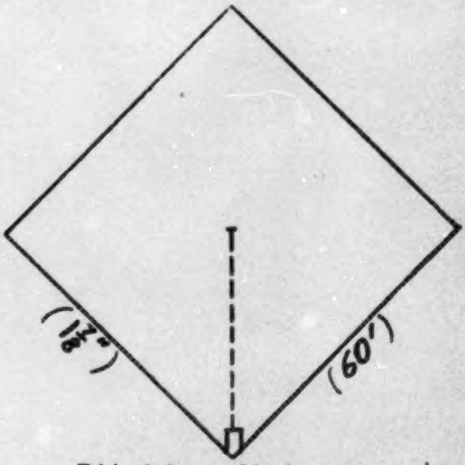
5.



6. Each angle is $128\frac{4}{7}$ degrees.

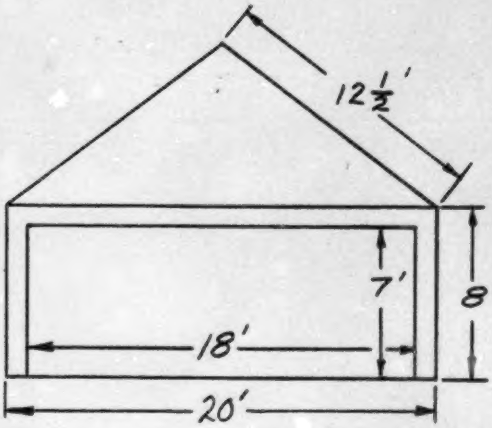

$128\frac{4}{7}°$
$\frac{1}{2}''$

II. 1. $\frac{1}{32}'' = 1'$

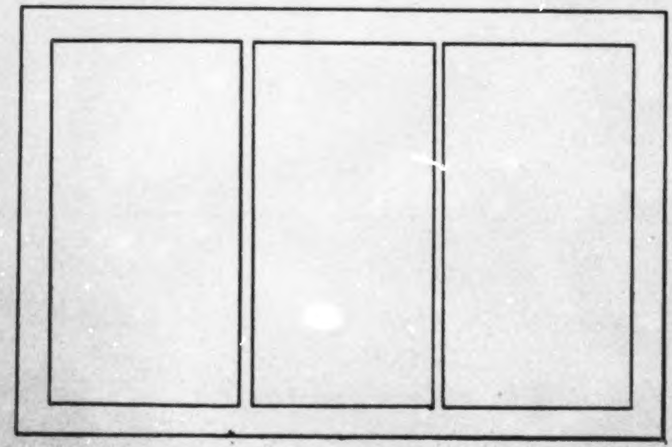


Pitching distance ≈ 41 ft.

II. 2. $\frac{1}{8}'' = 1'$

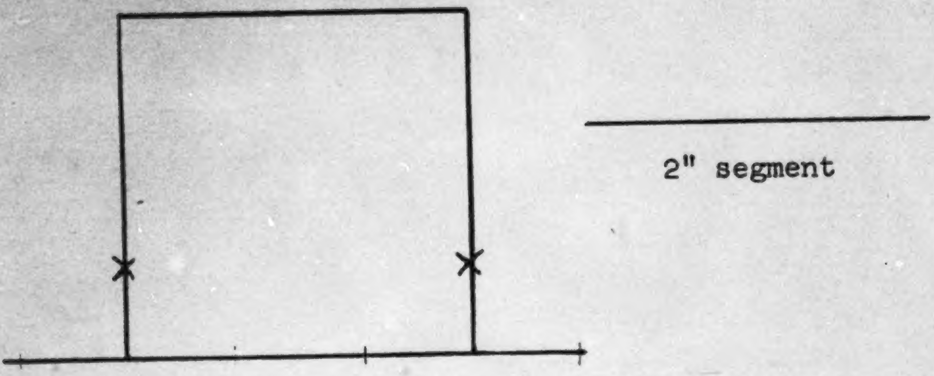

$12\frac{1}{2}'$
7'
8'
18'
20'

II. 3. $\frac{1}{16}'' = 1'$



III.

1.



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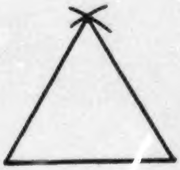

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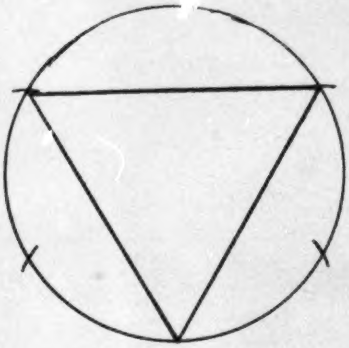

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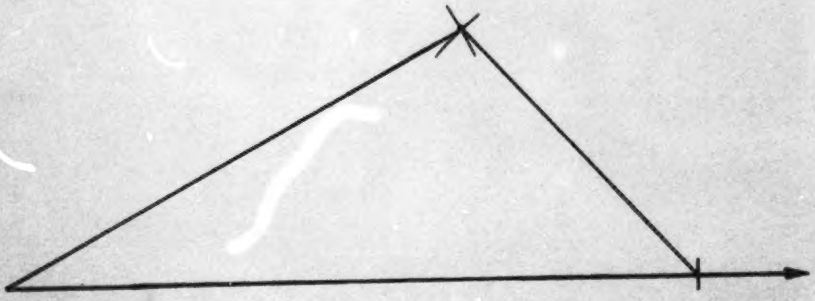
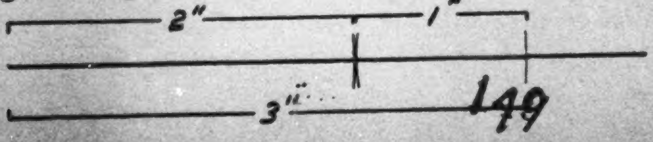
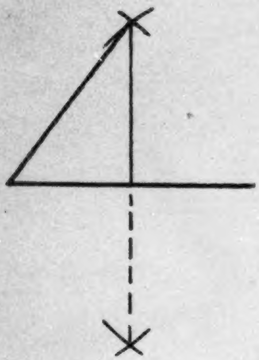


5. 1 + 2 = 3. The sum of the lengths of the two shorter segments must be greater than the length of the largest segment.



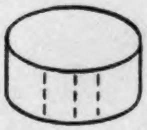
6. Answers will vary.
7. Answers will vary.
8.



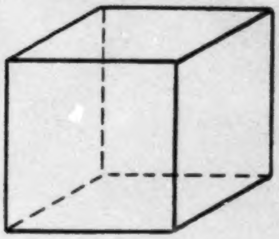
(There are other ways that this can be done. This is the easiest.)

IV.

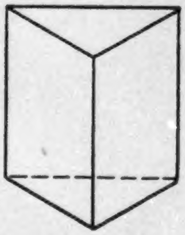
1.



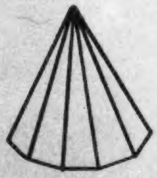
2.

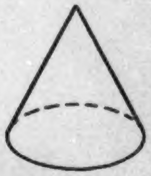


3.



4.





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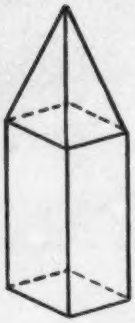
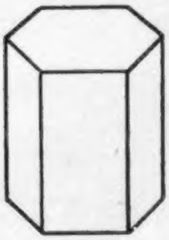
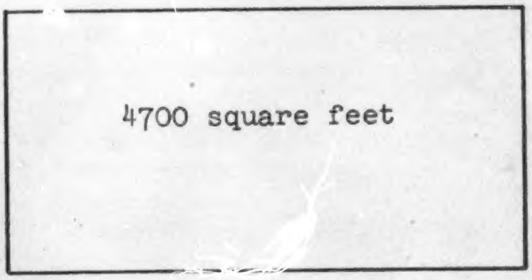


6.



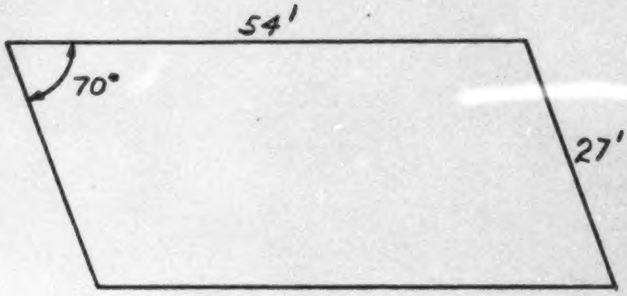
V.

1. $\frac{1}{32}'' = 1'$


4700 square feet

2. 600 miles
3. $4\frac{1}{6}''$

4. 1" = 18 feet

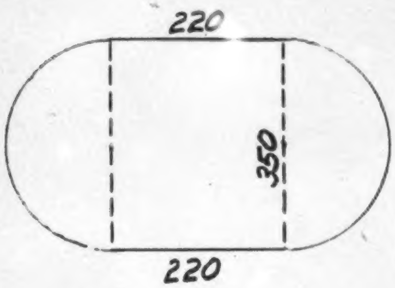


height $\approx 25\frac{3}{8}'$

perimeter $\approx 162'$

area $\approx$ 1370 square feet (Using $25\frac{3}{8}$, it is $1370\frac{1}{4}$ square feet.)

5. This does not need to be a scale drawing.



$$\begin{aligned} p &\approx 220 + 220 + \pi d \\ &\approx 440 + 350 \cdot \frac{22}{7} \\ &\approx 1540 \text{ yards} \end{aligned}$$

1 mile = 1760 yards

so the track is 220 yards less than one mile. One lap plus an additional straightaway is equal to one mile.